PG 15 Ross

(1) $S = \{(R,R), (R,G), (R,B), (G,R), (G,G), (G,B), (B,R), (B,G), (B,B)\}$

$|S| = 9$

prob each pt in sample space $= \frac{1}{9}$

(2) $S = \{(R,G), (R,B), (G,R), (G,B), (B,R), (B,G)\}$

$|S| = 6$

prob. each pt in sample space $\frac{1}{6}$

(3) $S = \cancel{\{(H,H), (H,T,H), (H,T,T,H), (H,T,T,T,H)}$

$\{(H,H), (T,H,H), (T,T,H,H), \ldots (\underbrace{T,T,\ldots T}_{n\ T's}, H,H) \ldots\}$

$\quad 0 \qquad 1 \qquad 2$

$\frac{1}{4} \qquad \frac{1}{2^3} \qquad \left(\frac{1}{2}\right)^4$

~~prob~~ =

$$\sum_{i=2}^{\infty}\left(\frac{1}{2}\right)^i = \left(\frac{1}{2}\right)^2 \sum_{i=0}^{\infty}\left(\frac{1}{2}\right)^i = \frac{1}{4}\,\frac{1}{\left(1-\frac{1}{2}\right)} = \frac{1}{4}\,\frac{1}{\frac{1}{2}} = \frac{1}{2}$$

Why not 1 ?

$$S = \{(H,H),\ (T,H,H),\ (T,T,H,H),\ \ldots\ (T,T,\ldots,T,H,H)\}$$

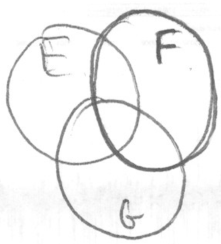
$$p \qquad \tfrac{1}{2}p \qquad \left(\tfrac{1}{2}\right)^2 p \qquad \ldots \qquad \left(\tfrac{1}{2}\right)^n p$$

$$\sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^k p = p\,\frac{1}{1-\frac{1}{2}} = \frac{p}{\frac{1}{2}} = 2p = 1$$

$$\Rightarrow p = \tfrac{1}{2}$$

$\therefore$ prob 4 tosses $= \left(\tfrac{1}{2}\right)^2 \cdot p = \left(\tfrac{1}{2}\right)^2 \cdot \tfrac{1}{2} = \tfrac{1}{8}$

4 (a)



$F - (F \cap E) - (F \cap G)$

$F \cap (\neg E) \cap (\neg G)$

(b) $E \cap F - G = E \cap F \cap (\neg G)$

(c) $E \cup F \cup G$

(d) $(E \cap F) \cup (E \cap G) \cup (F \cap G)$

(e) $E \cap F \cap G$

(f) $\neg(E \cup F \cup G)$

(g) E occurs not F & G

or F occurs not E & G

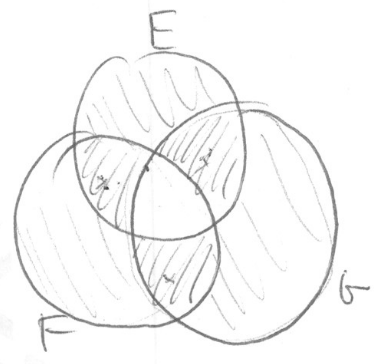
G occurs not E & F

$E \cap (\neg F \wedge - G)$

$\cup \quad F \cap (\neg E \wedge \neg G)$

$\cup \quad G \cap (\neg E \cap \neg F)$

(h)



$E \cup F \cup G - (E \cap F \cap G)$

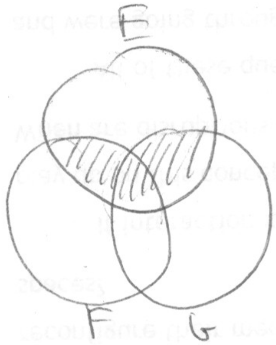
$= (E \cup F \cup G) \cap [(\neg E) \cup (\neg F) \cup (\neg G)]$

(5) $S = \left\{ \frac{1}{2}, \left(\frac{1}{2}\right)\left(\frac{1}{2}\right), \left(\frac{1}{2}\right)^2 \right\}$

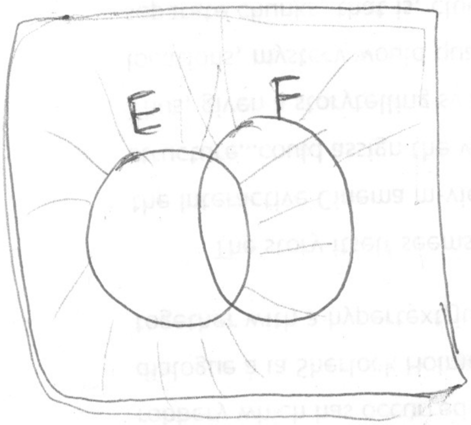
win on 1st bet — win on 2nd bet — loose both bets

$Pr\{\text{winner}\} = \frac{3}{4}$

(6) $E(F \cup G) = EF \cup EG$



(7) $(E \cup F)^c = E^c \cap F^c$



(8) $P(E) = .9 \quad P(F) = .8$

$P(EF) \geq .7$

$\cancel{E \cup F = Z}$

32 Pr[men 1 does not se

Pr[men 1 does not select his own hat, men 2 does not select his own hat, ...

... men n does not select his own hat]

$$= \Pr[M_1, M_2, \dots, M_n] = \Pr[M_1]\Pr[M_2 | M_1]\Pr[\quad \dots \quad]$$

$$\underbrace{\left(\frac{n-1}{n}\right)(\quad)}_{\text{product}}$$

M_i = men i ~~select~~ does not select his own hat

$\bar{M}_i$ = event men i does select his own hat.

Pr[none happens] = 1 − Pr[at least one happens]

Pr[At least one ~~happ~~ men selects his own hat]

$$= \Pr[\bar{M}_1 \cup \bar{M}_2 \cup \dots \cup \bar{M}_n] = \Pr[\bar{M}_1] + \Pr[\bar{M}_2] + \dots + \Pr[\bar{M}_n]$$

$$- \Pr[\bar{M}_1, \bar{M}_2]$$

$$= \sum_{i=1}^{n} \Pr[\bar{M}_i] - \sum_{i<j} \Pr[\bar{M}_i, \bar{M}_j] + \sum_{i<j<k} \Pr[\bar{M}_i, \bar{M}_j, \bar{M}_k]$$

...

$$= \Pr[\bar{M}_i] = \frac{1}{n} \; ; \qquad \Pr[\bar{M}_i, \bar{M}_j] = \left(\frac{1}{n}\right)\cdot\left(\frac{1}{n-1}\right)$$

$\left\{ \begin{array}{l} \binom{n}{0} = \frac{n!}{n!0!} = 1 \\ \binom{n}{n} = 1 \end{array} \right.$ 2

$$Pr[\bar{M}_i, \bar{M}_j, \bar{M}_k] = \left(\frac{1}{n}\right)\left(\frac{1}{n-1}\right)\cdot\left(\frac{1}{n-2}\right)$$

So $\therefore$ $Pr[\bar{M}_1, \bar{M}_2, \bar{M}_3, \ldots, \bar{M}_n] = \frac{1}{n!}$

$$Pr[\bar{M}_1 \cup \bar{M}_2 \cup \cdots \cup \bar{M}_n] = n\left(\frac{1}{n}\right) - \binom{n}{2}\frac{1}{n}\left(\frac{1}{n-1}\right) + \binom{n}{3}\frac{1}{n}\left(\frac{1}{n-1}\right)\left(\frac{1}{n-2}\right)$$

$$+ \cdots +$$

$$= \sum_{k=1}^{n} \binom{n}{k} \frac{(-1)^{k+1}}{n(n-1)\cdots\cancel{(n-k)}(n-k+1)} = \sum_{k=1}^{n} \frac{\cancel{n(n-1)(n-2)\cdots(n-k+1)}}{k!} \frac{(-1)^{k+1}}{\cancel{n(n-1)\cdots(n-k+1)}}$$

$$= \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k!}$$

So

$$Pr[\text{At least one man selects his own hat}] = 1 - \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k!}$$

$$= \sum_{k=2}^{n} \frac{(-1)^k}{k!} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \cdots + \frac{(-1)^n}{n!}$$

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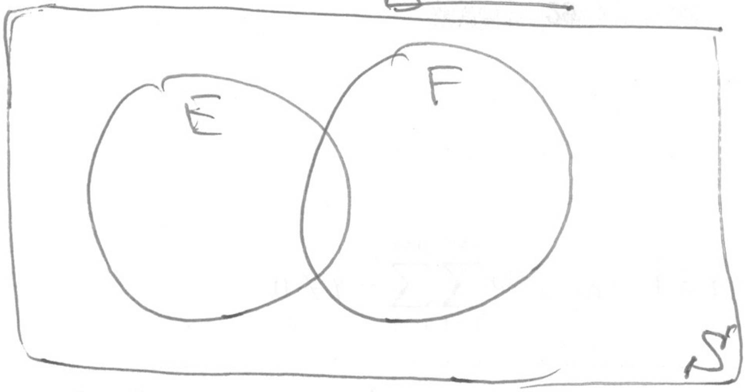
(8)



Since ~~E~~ $E \cup F \subset S$

$$P(E \cup F) \le 1 \Rightarrow 0 \ge P(E \cup F) - 1$$

Then ~~Adding~~ ~~[illegible]~~ Adding $P(E \cap F)$ to both sides gives

$$P(E \cap F) \ge \underbrace{P(E \cap F) + P(E \cup F)}_{} - 1$$

$$P(E) + P(F) - 1$$

$\therefore$ Bonferroni's Inequality results

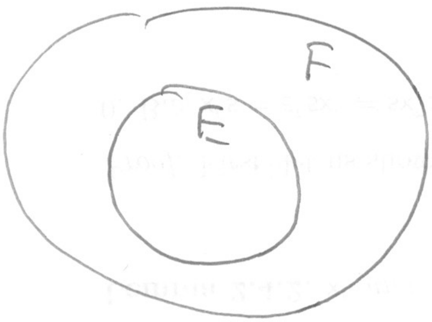
$$P(E \cap F) \ge P(E) + P(F) - 1$$

w/ $P(E) = .9$ & $P(F) = .8$ one gets

$$P(E \cap F) \ge .9 + .8 - 1 = .7$$ ✓

(9) If $E \subset F$

$$P(F) = P(E) + P(F \cap E^c) \geq P(E)$$



$\therefore$ $F = E \cup (F \cap E^c)$ + since E + $F \cap E^c$ are disjoint

the result follows.

(10) Show:

$$P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum_{i=1}^{n} P(E_i) \quad *$$

Eq: 1.2 is $P(E \cup F) = P(E) + P(F) - P(EF) \leq P(E) + P(F)$

So $*$ holds for $n = 2$.

Assume $*$ holds for $n = k$.

+ Consider

$$P\left(\bigcup_{i=1}^{k+1} E_i\right) = P\left(E_{k+1} \cup \left(\bigcup_{i=1}^{k} E_i\right)\right) \leq P(E_{k+1}) + P\left(\bigcup_{i=1}^{k} E_i\right)$$

$$= \sum_{i=1}^{k+1} P(E_i)$$

Alternative method: let

$$F_1 = E_1 \quad \& \quad F_i = E_i \prod_{j=1}^{i-1} E_j^c$$

so $$F_2 = E_2 \prod_{j=1}^{1} E_j^c \; , \quad F_3 = E_3 \prod_{j=1}^{2} E_j^c$$

Then $$\bigcup_{i=1}^{n} E_i = \bigcup_{i=1}^{n} F_i \quad \& \quad F_i \cap F_j = \phi \;\; \forall\, i \neq j \text{ i.e.}$$

F_i & F_j are disjoint, thus

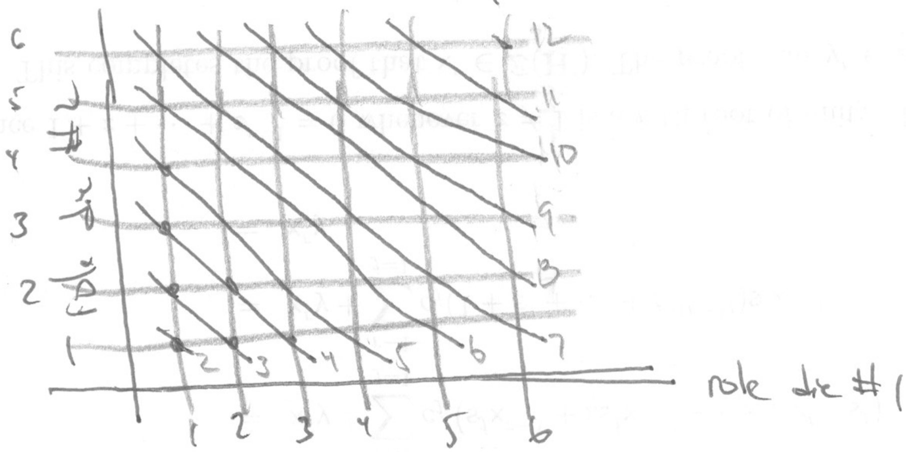
$$P\left(\bigcup_{i=1}^{n} E_i\right) = P\left(\bigcup_{i=1}^{n} F_i\right) = \sum_{i=1}^{n} P(F_i) \qquad \text{By disjointness}$$

& each $F_i \subset E_i$ so

$$\leq \sum_{i=1}^{n} P(E_i)$$

∎

11



$$P(S=2) = \frac{1}{36} = P(S=12)$$

$$P(S=3) = \frac{2}{36} = P(S=11)$$

$$P(S=4) = \frac{3}{36} = P(S=10)$$

$$P(S=5) = \frac{4}{36} = P(S=9)$$

$$P(S=6) = \frac{5}{36} = P(S=8)$$

$$P(S=7) = \frac{6}{36}$$

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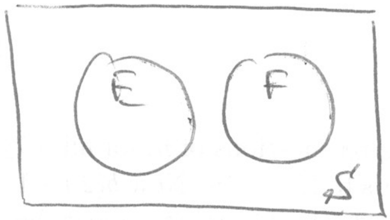
(12)



Sample space is $E, F, (E \cup F)^c$

$P(E), P(F), 1 - P(E) - P(F)$

Then what is the prob. that E occurs before F?

If we perform the original exp. n times the prob that E occurs on the n-th time 1st is

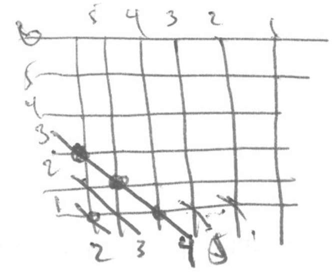
$P(E)(1-p)^{n-1}$ or geometrically distributed

$$\sum_{n \geq 1} P(E)(1-p)^{n-1}$$ is the prob. that E happens at least once

$$= P(E) \sum_{n \geq 0} (1-p)^n = P(E) \frac{1}{1-(1-p)} = \frac{P(E)}{P(E)+P(F)}$$

Don't follow?

(13)



P(win) = P(throw 7) + P(throw 11)

+ P(mks pt | throws 4, 5, 6, 8, 9, 10) P(throws 4, 5, 6 ...)

on 1st toss

P(mks pt | throws 4) =

$P_4 = \frac{3}{36}$ $P_7 = \frac{6}{36}$ from problem 12 prob event 4 occurs before event 7 is given by $\frac{3/36}{6/36} = 3/6 = \frac{1}{2}$

$\therefore$ P(mks pt | throws 4) = $\frac{1}{2}$

P(mks pt | throws 5) = $\frac{4}{6} = \frac{2}{3}$

P(mks pt | throws 6) = $\frac{5}{6}$

P(mks pt | throws 8) = $\frac{5}{6}$

p(mks pt | throws 9) = $\frac{4}{6} = \frac{2}{3}$

p(mks pt | throws 10) = $\frac{3}{6} = \frac{1}{2}$

$$P(\text{win}) = \frac{6}{36} + \frac{2}{36} + \sum_{i=\{4,5,6,8,9,10\}} P(\text{mks pt} \mid \text{throws } i)\, P(\text{throw } i)$$

$$= \frac{8}{36} + \underset{i=4}{\frac{3}{36}\left(\frac{1}{2}\right)} + \underset{i=5}{\frac{4}{36}\left(\frac{2}{3}\right)} + \underset{i=6}{\frac{5}{36}\left(\frac{5}{6}\right)} + \underset{i=8}{\frac{5}{36}\left(\frac{5}{6}\right)}$$

$$+ \underset{i=9}{\frac{4}{36}\left(\frac{2}{3}\right)} + \underset{i=10}{\frac{3}{36}\left(\frac{1}{2}\right)}$$

2, 3, 7, 11, 12

4, 5, 6, 8, 9, 10

$$P(\text{win}) = \frac{8}{36} + \cancel{\frac{4}{36}} \left[\frac{3}{36}\left(\frac{3}{6}\right) + \frac{4}{36}\left(\frac{4}{6}\right) + \frac{5}{36}\left(\frac{5}{6}\right) + \frac{5}{36}\left(\frac{5}{6}\right) \right.$$

$$\left. + \frac{4}{36}\left(\frac{4}{6}\right) + \frac{3}{36}\left(\frac{3}{6}\right) \right)$$

$$= \frac{8}{36} + \cancel{\frac{4}{36}} \frac{1}{6 \cdot 36}\left[\underbrace{9 + 16 + 25 + 25 + 16 + 9}_{100} \right]$$

$$= .68\ldots = \frac{37}{54} \quad \#$$

(14) (A) (B)

$$P(A \text{ wins}) = p + (1-p)^2 p + (1-p)^2(1-p)^2 p + \cdots + (1-p)^{2k} p + \cdots$$

$$P(B \text{ wins}) = (1-p)p + (1-p)^2(1-p)p + \cdots + (1-p)^{2k+1} p$$

$$P(A \text{ wins}) = \sum_{k \geq 0} \left[(1-p)^2\right]^k p = p \frac{1}{1-(1-p)^2} = \frac{p}{1-1+2p-p^2} = \frac{p}{-p^2+2p}$$

$$= \frac{1}{-p+2} = \frac{1}{2-p}$$

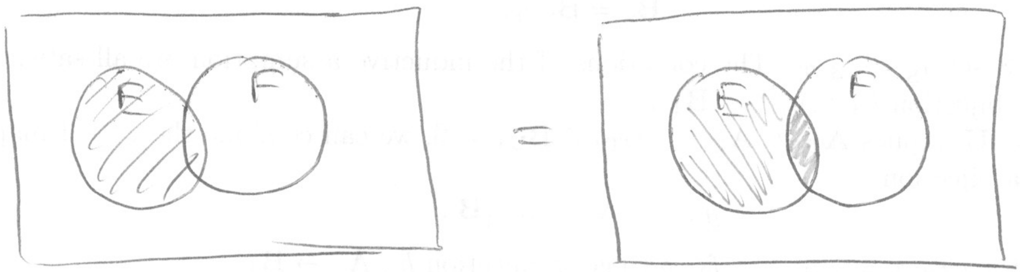
$$P(B \text{ wins}) = \sum_{k \geq 0} (1-p)^{2k+1} p = (1-p)p \sum_{k \geq 0} (1-p)^{2k} = p(1-p) \frac{1}{1-(1-p)^2}$$

$$= \frac{p(1-p)}{(1-(1-p))(1+(1-p))} = \frac{\cancel{p}(1-p)}{\cancel{p}(2-p)} = \frac{1-p}{2-p}$$

Check $P(A) + P(B) \stackrel{?}{=} 1$ yes ✓

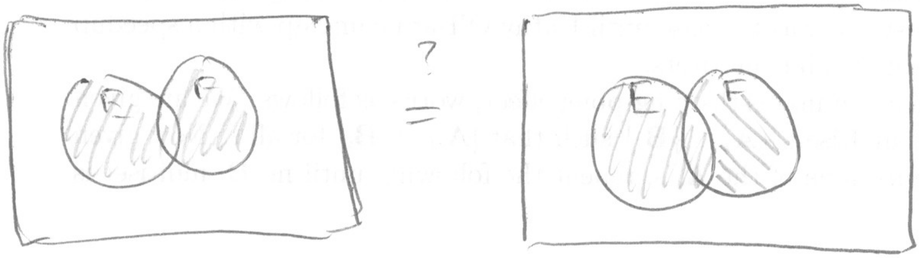
~~A~~ ~~p~~ $1 > 1-p$ $\therefore$ Always try to go 1st.

(15) $E = EF \cup EF^c = E \cap (F \cup F^c) = E$ ✓



union of disjoint sets

$E \cup F = E \cup FE^c$



union of disjoint sets

(16) ~~E~~ From $E = EF \cup EF^c$ since EF & EF^c are disjoint

$P(E) = P(EF) + P(EF^c)$

& From $E \cup F = E \cup FE^c$

$P(E \cup F) = P(E) + P(FE^c)$

~~For EF~~ $F = (F \cap E) \cup (F \cap E^c)$

$\therefore \quad P(F) = P(FE) + P(FE^c)$

$\therefore \quad P(E \cup F) = P(E) + P(F) - P(FE)$

D

A	B	C
$\{H,T\}$	$\{H,T\}$	$\{H,T\}$

size of sample space is

$2^3 = 8$

Game ends w/ ~~[illegible]~~

H	H	T
T	T	H
	$\vdots$	
	etc	

$$p(\text{ends}) = \binom{3}{1}p^2(1-p) + \binom{3}{1}p(1-p)^2$$

$$= 1 - p^3 - (1-p)^3$$

P(ends 1st ~~tos~~ tosses) $= 3p^2(1-p) + 3p(1-p)^2$

$p = 1/2 \qquad 3\left(\frac{1}{4}\right)\frac{1}{2} + 3\left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = \frac{3}{4}$

v.s. $\qquad 1 - \frac{1}{8} - \frac{1}{8} = \frac{3}{4}$

(18) $P(\text{Both children are girls} \mid \text{1st child} = \text{girl}) = ?$

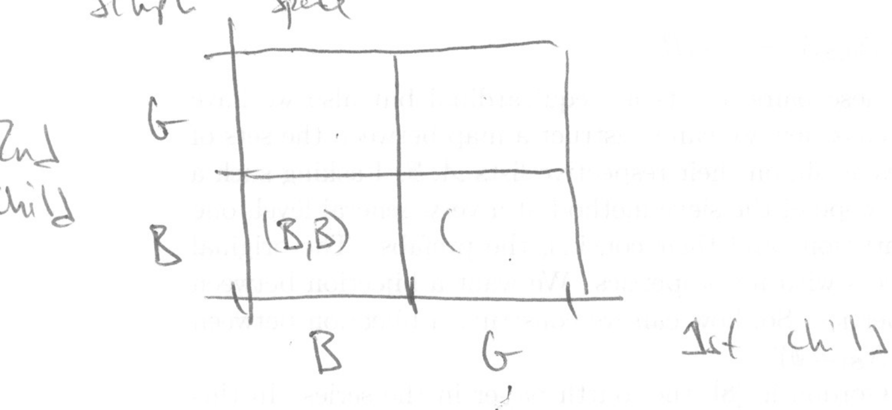
$P(\text{1st child} = \text{girl} \mid \text{Both chil}) = 1$

$P(\cancel{G_1 = G \mid G_2 = G})$

$$\frac{P(\text{1st child} = G \mid \text{Both are} = \text{Girls})\, P(\text{Both are Girls})}{P(\text{1st child girl})}$$

$$= \frac{1(1/4)}{1/2} = \frac{1}{2}$$

Sample space



a) $P(\text{2nd Child} = G \mid \text{1st Girl}) = 1/2$

b) $P(\text{Both children are girls} \mid \text{At least 1 is a girl})$

$$= \frac{1\,(P(\text{Both children are girls}))}{P(\text{at least 1 is a girl})} = \frac{1/4}{3/4} = \frac{1}{3}$$

(19)

$\cancel{P(\text{one face is } 6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}}$

Bernoulli trial (to get 6) w/ # trials = 2

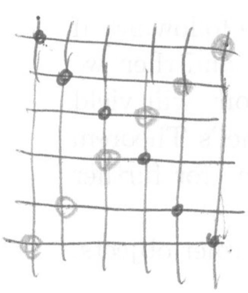
$$P(n=1) = \binom{2}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^1 = \frac{10}{36}$$

$$+\,P(n=2) = 1\left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(\text{at least } 1\ 6) = \frac{11}{36}$$

$$P(\text{at least } 1\ 6 \mid \text{two faces are different}) = \frac{P(\text{at least } 1\ 6\ \&\ \text{two faces different})}{P(\text{two faces different})}$$



$$= \frac{\cancel{11/36}\ 10/36}{\frac{36-6}{36}} = \frac{10}{30} = \frac{1}{3}$$

(20) A B C Sample space = 6^3

x y z

$$P(x=y \cup x=z \cup y=z) = 3P(x=y) = \frac{1}{2}$$

$$\underbrace{\sum Pr(A=n, B=m, C=l)}$$

$x=y\neq z$ $\left(\frac{1}{6}\right)^3$

$x\neq y=z$

$x=z\neq y$

$$P(\text{All distinct \#'s}) = \left(\frac{5}{6}\right)\left(\frac{4}{6}\right)$$

? not sure

(21) $p_m = .05$

$p_w = .0025$

$$p(\text{person}=\text{male} \mid \text{colorblind}) = \frac{p(\text{colorblind} \mid \text{male})P(\text{male})}{p(\text{colorblind})}$$

Here $\boxed{p(\text{colorblind} \mid \text{male}) = .05}$

$$= \frac{(.05)(1/2)}{p(\text{colorblind} \mid \text{male})P(\text{male}) + p(\text{colorblind} \mid \text{female})P(\text{female})}$$

$$= \frac{(.05)(1/2)}{(.05)(1/2) + (.0025)(1/2)} = \cdots$$

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$p_A = p$

$$X_i = \begin{cases} 0 & A \text{ looses point } i \\ 1 & A \text{ wins point } i \end{cases}$$

$$p(X_i) = \begin{cases} 1-p & X_i = 0 \\ p & X_i = 1 \end{cases}$$

$p(\text{total \# pts} = 2) = p^2$

$p(\text{total \# pts} = 4) = p(1-p)p \qquad = \sum_k \binom{4}{k} p^{k} (1-p)^{4-k}$

1	2	3	4
A	A		
A	B		
B	A		

A B ~~B~~ A

$\boxed{A\ B\ B\ B}$ ✓ $p(1-p)^3$

A B ~~A~~ B

A B A A ✓ $p^3(1-p)$

$$\sum \cancel{\binom{4}{k_1}} p^{k_1} (1-p)^{k_2}$$

$k_1 + k_2 = 4 \quad \Rightarrow \quad \# \text{ trials} = 4$

$|k_1 - k_2| = 2 \Rightarrow$ one opponent wins by 2 pts

$k_2 = 4 - k_1$

$$\sum p^k (1-p)^{4-k}$$

$\Rightarrow$ $$\underbrace{|k-(4-k)|}=2 \Rightarrow$$

$$|2k-4| = 2$$

$$|k-2| = 1$$

$k = 3, 1$

$$p(\#pts = 2n) = \sum_{\underbrace{|k-2n+k|=2}} \binom{2n}{k} p^k (1-p)^{2n-k}$$

$$|2k-2n| = 2$$

$$\Rightarrow |k-n| = 1$$

$\Rightarrow$ ~~$|k-2n+k|=2n$~~

$$k = n+1 \; \& \; k = n-1$$

$$p(\#pts = 2n) = \binom{2n}{n+1} p^{n+1} (1-p)^{n-1} + \binom{2n}{n-1} p^{n-1} (1-p)^{n+1}$$

\#

total ~~of~~ n pts

$$\cancel{P(A \text{ wins})} = P(A \text{ wins w/ game to } 2n \text{ pts}) = \binom{2n}{n+1} p^{n+1} (1-p)^{n-1}$$

$$P(A \text{ wins}) = \sum_{n=1}^{\infty} \binom{2n}{n+1} p^{n+1} (1-p)^{n-1}$$

How sim?

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$$P(E_1(E_2 \cdots E_n)) = P(E_1) P(E_2 \cdots E_n | E_1)$$

$$= P(E_1) P(E_2(E_3 \cdots E_n) | E_1) = P(E_1) P(E_2 | E_1) P(E_3 \cdots E_n | E_1 E_2)$$

$$= \cdots P(E_1) P(E_2 | E_1) P(E_3 | E_1 E_2) \cdots P(E_n | E_1 E_2 \cdots E_{n-1})$$

(24) $A \leftarrow n$ votes

$B \leftarrow m$ votes $\qquad n > m$

$$\underbrace{\_\ \_\ \_\ \_\ \cdots\ \_\ \_\ \_\ \_}$$

$n+m$ slots n of which must be ~~tstb~~ labeled w/ An A the remeiny B of which

(a) $P_{2,1}$ = prob A always in the lead

$\underline{A}, \underline{\ \ }, \underline{\ \ } \rightarrow A, B, A$

$\underline{B}, \underline{A}, \underline{A} \quad \searrow A, A, B$

$\therefore$ sample space is

$$\{(A,B,A), (A,A,B), (B,A,A)\} \qquad P_{2,1} = \frac{2}{3} \text{ or } \frac{1}{3} \text{ if ties are not counted}$$

(b) $P_{3,1} = ?$

$$S = \{(B,A,A,A), (A,B,A,A), (A,A,B,A), (A,A,A,B)\}$$

$$P_{3,1} = \frac{2}{4} = \frac{1}{2} \quad \text{if ties are not count}$$

$=$

(c) $P_{n,1} = ?$ $\quad S = \{(B,A,\dots),(A,B,\dots),(A,A,B,\dots)\dots\}$

$$P_{n,1} = \frac{n-1}{n+1} = \frac{n+1-2}{n+1} = 1 - \frac{2}{n+1}$$

(d) $P_{3,2} = ?$ $\qquad \binom{5}{2} = \frac{5\cdot 4}{2} = 10$

$$S = \{(\underline{B},\underline{B},\_,\_,\_),(\underline{B},\_,\underline{B},\_,\_),(\underline{B},\_,\_,\underline{B},\_),(\underline{B},\_,\_,\_,\underline{B}),$$
$$(\_,\underline{B},\underline{B},\_,\_),(\_,\underline{B},\_,\underline{B},\_),(\_,\underline{B},\_,\_,\underline{B}),$$
$$(\_,\_,\underline{B},\underline{B},\_),(\_,\_,\underline{B},\_,\underline{B}),(\_,\_,\_,\underline{B},\underline{B})\}$$

$$P_{3,2} = \frac{3}{10}$$

(e) $P_{4,2} = \frac{(n-1)}{15}$

$n+1-2 = n-1$

n dots (7)

m lines (3)

n+1 possible spaces for lines

Require that (1) $m<n$

(2) 1st

$$\binom{6}{2} = \frac{6\cdot 5}{2} = 15$$

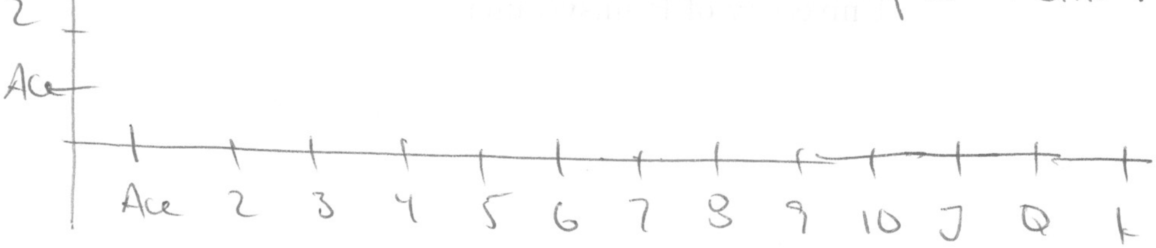
? Don't see

(25)

(a) $\frac{3}{51}$

(b) $P(\text{Pair} \mid \text{suits different}) = \dfrac{P(\text{Pair} \cap \text{suits diff})}{P(\text{suits diff})}$

$= \dfrac{P(\text{pair})}{P(\text{suits differ})}$



$= \dfrac{3/51}{1 - P(\text{suits same})} = \dfrac{3/51}{1 - 12/51} = \dfrac{3/51}{39/51} = \frac{3}{39} = \frac{1}{13}$

$\begin{array}{r} \overset{4}{\cancel{5}}1 \\ 12 \\ \hline 39 \end{array}$

pg 18 Ross

(26) $P(E_1E_2E_3E_4) = P(E_1)P(E_2E_3E_4 | E_1)$

$= P(E_1)P(E_2|E_1)P(E_3E_4 | E_1E_2)$

$= P(E_1)P(E_2|E_1)P(E_3|E_1E_2)P(E_4|E_1E_2E_3)$

$P(E_1) = \dfrac{\binom{4}{1}}{\binom{52}{13}}$

$P(E_2|E_1) = \dfrac{\binom{3}{1}}{\binom{39}{13}}$ $52-13=39$

$P(E_3|E_1E_2) = \dfrac{\binom{2}{1}}{\binom{26}{13}}$ $39-13=26$

$P(E_4|E_1E_2E_3) = 1$?

27 Note $\cancel{E_1 \supset E_2}$ $P(A|B)$

$\Leftarrow B \supset A$

$\cancel{E_4} = E_1 \supset E_2 \supset E_3 \supset E_4$

So $P(E_1E_2E_3E_4) = P(E_4)P(E_4|E_1E_2E_3)$

$P(E_1) = 1$

$P(E_2) =$

28 $P(A|B) > P(A)$

$\frac{P(B|A)P(A)}{P(B)} = \frac{P(AB)}{P(B)} > P(A)$ $P(B|A) \overset{?}{\geq} P(B)$

$\Rightarrow P(B|A) > P(B)$ Yes

29 (a) $P(E|F) = 0$

(b) $P(E|F) > .6$ since $P(E|F) = \frac{P(EF)}{P(F)} = \frac{P(E)}{P(F)}$

(c) $P(E|F) = 1$ by sim reason

$= \frac{.6}{P(F)} > .6$

30 $P(B) = .7$ $P(\text{Hit} | \text{Bill}) = .7$

$P(G) = .4$ $P(\text{Hit} | \text{George}) = .4$

(a) $P(\cancel{\text{Hit}}) \cancel{= ? = \sum_{\text{person}} P(\text{Hit}, \text{Person})}$

Hit = T $\quad P(\text{George} \mid \text{Hit}=T) = \dfrac{P(\text{Hit} \mid \text{George})\, P(\text{George})}{P(\text{Hit}=T)}$

Assume
$P(\text{Person}=i) = 1/2$

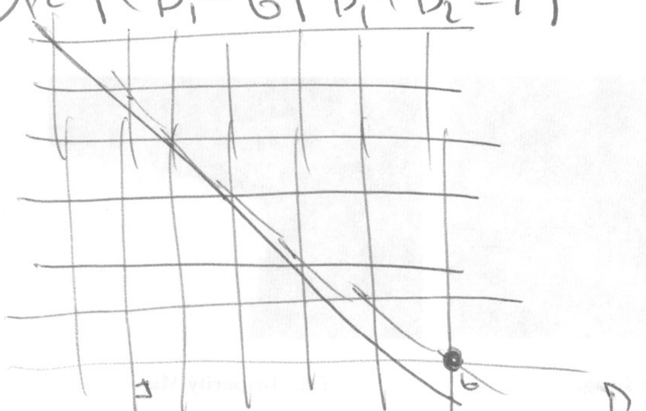
$$= \frac{P(\text{Hit} \mid \text{George})\, P(\text{George})}{\sum_i P(\text{Hit}=T \mid \text{Person } i)\, P(\text{Person } i)}$$

$$= \frac{(.4)(1/2)}{.7(1/2) + .4(1/2)} = \frac{.4}{.7+.4} = -$$

(b) ?

(31) $P(D_1 = 6 \mid D_1 + D_2 = 7) = \dfrac{P(D_1 = 6 \;\&\; D_1 + D_2 = 7)}{P(D_1 + D_2 = 7)} = \dfrac{1/36}{7/36}$

$$= \frac{1}{7}$$



(33)

4 FB
6 FG
6 SB
X SG

$$\text{prob}(\text{sex} = \text{Boy}) = \frac{10}{16+x}$$

$$\text{prob}(\text{sex} = \text{Girl}) = \frac{6+x}{16+x}$$

$$\text{prob}(\text{class} = \text{fresh}) = \frac{10}{16+x}$$

$$\text{prob}(\text{class} = \text{soph}) = \frac{6+x}{16+x}$$

independent $\Rightarrow$ $\cancel{\frac{10}{16+x}} =$

$$\frac{10}{16+x} = \frac{6+x}{16+x} \Rightarrow x = 4$$

(34) Ex. Gamblers fallacy prob is the same each time

(35)

(a) $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

(b) $\frac{1}{16}$

(c) $1/2$

(36)

$\underline{B_1}$ $\qquad$ $\underline{B_2}$

$P_b = \frac{1}{2}$ $\qquad$ $P_b = \frac{2}{3}$

$P_w = \frac{1}{2}$ $\qquad$ $P_w = \frac{1}{3}$

$p(\text{Black} \mid B_1) = \frac{1}{2}$ $\qquad$ $p(\text{Black} \mid B_2) = \frac{2}{3}$

$p(\text{white} \mid B_1) = \frac{1}{2}$ $\qquad$ $p(\text{white} \mid B_2) = \frac{1}{3}$

$$P(\text{Black}) = P(\text{Black} \mid B_1)\, P(B_1) + P(\text{Black} \mid B_2)\, P(B_2)$$

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{2}{3}\right) = \frac{1}{2}\,\frac{1}{6}(3+4) = \frac{7}{12}$$

$\Rightarrow \quad P(\text{white}) = \frac{5}{12}$

(37) $P(B_1 \mid \text{White}) = \dfrac{P(\text{White} \mid B_1)}{P(\text{White})} = \dfrac{P(\text{White} \mid B_1)}{P(W \mid B_1)P(B_1) + P(W \mid B_2)P(B_2)}$

$$= \frac{1/2}{5/12} =$$

pg 19 Probs

(37)

$\underline{B_1}$

$P_w = \frac{1}{2}$

$P_b = \frac{1}{2}$

$\underline{B_2}$

$P_w = \frac{1}{3}$

$P_b = \frac{2}{3}$

$$Pr[B_1 | W] = ? = \frac{Pr[W | B_1] \cdot P[B_1]}{P[W]} = \frac{Pr[W|B_1]Pr[B_1]}{P[W|B_1]P[B_1] + P[W|B_2]P[B_2]}$$

$$= \frac{1/2(1/2)}{1/2(1/2) + 1/3(1/2)} = \frac{1/2}{1/2 + 1/3}$$

(38)

$\underline{U_1}$

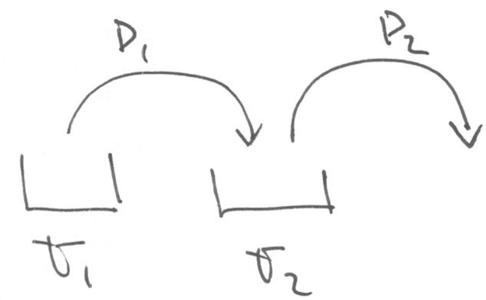
$P_W = \frac{2}{3}$

$P_b = \frac{1}{3}$

$\underline{U_2}$

$P_W = \frac{1}{6}$

$P_B = \frac{5}{6}$



$$Pr[\cancel{D_1 = W}] = ?$$

$$Pr[D_1 = W \mid D_2 = W] = ?$$

$$= \frac{Pr[D_2 = W \mid D_1 = W] Pr[D_1 = W]}{Pr[D_2 = W]}$$

$$= \frac{Pr[D_2=W \mid D_1=W]\,Pr[D_1=W]}{Pr[D_2=W \mid D_1=W]\,Pr[D_1=W] + Pr[D_2=W \mid D_1=B]\,Pr[D_1=B]}$$

$$Pr[D_2=W \mid D_1=W] = \frac{2}{7}$$

$$Pr[D_1=W] = \frac{2}{3} \qquad Pr[D_1=B] = \frac{1}{3}$$

$$\cancel{Pr[D_2=B] =}$$

$$Pr[D_2=W \mid D_1=B] = \frac{1}{7}$$

$$\therefore\ Pr[D_1=W \mid D_2=W] = \frac{(2/7)(2/3)}{(2/7)(2/3) + (1/7)(1/3)}$$

(39) 1

$$Pr[S=C_1 \mid W] = \frac{Pr[W \mid C_1]\, Pr[C_1]}{P[W]}$$

$Pr[W \mid C] = .7$ $\therefore$ $Pr[W] = Pr[W \mid A]P[A] + P[W \mid B]P[B]$

$Pr[W \mid A] = .5$ $+ P[W \mid C]P[C]$

$Pr[W \mid B] = .6$

$$P[A] = \frac{50}{50+75+100} =$$

$$P[B] = \frac{75}{50+75+100} =$$

$$P[C] = \frac{100}{50+75+100} = \ldots$$

(40) (a) C_1, C_2

$P(H \mid C_1] = 1/2$ $P[H \mid C_2] = 1$ $\rightarrow$ $P[H] = (1/2)(1/2) + 1(1/2)$

$= 3/4$

$$P[C_1 \mid H] = \frac{P[H \mid C_1]\, P[C_1]}{P[H]} = \frac{(1/2)(1/2)}{3/4} = \frac{1}{3}$$

(b) Desire

$$P[C_1 | \{H,H\}] = \frac{P[\{H,H\} | C_1] P[C_1]}{P[\{H,H\}]}$$

$$P[\{H,H\}] = P[\{H,H\} | C_1] P[C_1] + P[\{H,H\} | C_2] P[C_2]$$

$$= \left(\tfrac{1}{4}\right)\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{2}\right) = \tfrac{1}{8} + \tfrac{1}{2} = \tfrac{1}{8} + \tfrac{4}{8} = \tfrac{5}{8}$$

$$P[C_1] = \tfrac{1}{2}$$

$$P[\{H,H\} | C_1] = \tfrac{1}{4}$$

$$\therefore P[C_1 | \{H,H\}] = \frac{(1/4)(1/2)}{5/8} = \frac{1}{5}$$

(c) ~~By~~ $P[C_1] = 1$ since the false ~~[illegible]~~ coin could not show any tails. Mathematically

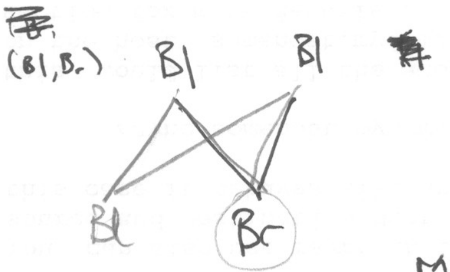
$$P[C_1 | \{H,H,T\}] = \frac{P[\{H,H,T\} | C_1] P[C_1]}{P[\{H,H,T\}]}$$

$$P_r[\{H,H,T\}] = \underbrace{P[\{H,H,T\} | C_1]}_{\left(\frac{1}{8}\right)\left(\frac{1}{2}\right)} P[C_1] + \underbrace{P[\{H,H,T\} | C_2]}_{=0} P[C_2]$$

$\cancel{P[\{H,H,T\}]}$

$$P[C_1 \mid \{H,H,T\}] = 1$$

(41)



	Mother	Father	
possible configurations?	(Bl, Br)	(Bl, Br)	only.
	~~(Bl, B)~~		

Mother \ Father	Bl	Br
Bl	1/4	1/4
Br	1/4	1/4

(a) $P[(Bl,Bl)] = \frac{1}{4}$

(b) $P[(Bl,Bl) \mid \text{Parent} = \text{Brown}]$

$P[\text{Child of this rat w/ Brown partner}] = \text{Black}$

?

(42) $P[H|C_1] = 1 \quad P[H|C_2] = \frac{1}{2} \quad P[H|C_3] = .75$

$$P[C_2|H] = \frac{P[H|C_2]P[C_2]}{P[H]} \quad \ldots$$

(43) $P[H|C_i] = \frac{i}{10}$

$$P[C_5|H] = \frac{P[H|C_5]\,P[C_5]}{\sum_{i=1} P[H|C_i]P[C_i]}$$

(44)

$\underline{U_1}$	$\underline{U_2}$
$P_W = \frac{5}{12}$	$P_W = \frac{3}{15}$
$P_B = \frac{7}{12}$	$P_B = \frac{12}{15}$

$$P[\text{Tail}\,|\,W] = P[U_2|W] = \frac{P[W|U_2]\,P[U_2]}{\sum_{i=1}^{2} P[W|U_i]P[U_i]}$$

Tail

$$= \frac{(3/15)(1/2)}{(5/12)(1/2) + (3/15)(1/2)}$$

5

(45) $$P[D_1=B \mid D_2=R] = \frac{P[D_2=R \mid D_1=B]\,P[D_1=B]}{P[D_2=R]}$$

$$P[D_1=B] = \frac{b}{b+r}$$

$$P[D_2=R \mid D_1=B] = \frac{r}{r+(b+c)}$$

$$P[D_2=R] = P[D_2=R \mid D_1=B]\,P[D_1=B] + P[D_2=R \mid D_1=R]\,P[D_1=R]$$

$$= \frac{r}{r+(b+c)}\left(\frac{b}{b+r}\right) + \frac{r+c}{(r+c)+b}\left(\frac{r}{r+b}\right)$$

$$P[D_1=B \mid D_2=R] = \frac{1}{1 + \dfrac{P[D_2=R \mid D_1=R]\,P[D_1=R]}{P[D_2=R \mid D_1=B]\,P[D_1=B]}}$$

$$= \frac{1}{1 + \dfrac{\frac{r+c}{\cancel{(r+c)+b}}\left(\frac{\cancel{r}}{\cancel{r+b}}\right)}{\frac{\cancel{r}}{\cancel{r+(b+c)}}\left(\frac{b}{\cancel{b+r}}\right)}} = \frac{1}{1+\dfrac{r+c}{b}}$$

$$= \frac{b}{b+r+c}$$ ✓

(46) A, B, C 6

$P[A] =$ prob A is executed $\bar{A} =$ A is Not ~~executed~~ executed

$$P[A|\bar{B}] = \frac{P[\bar{B}|A]\,P[A]}{P[\bar{B}]}$$

$$= \frac{1(1/3)}{P[\bar{B}|A]P[A] + P[\bar{B}|C]P[C]} = \frac{1/3}{1(1/3)+1(1/3)} = \frac{1/3}{2/3} = \frac{1}{2}$$

seems wrong?