Y \ X	1	2	3	4	$P_Y(y)$
1	1/8	1/16	1/32	1/32	1/4
2	1/16	1/8	1/32	1/32	1/4
3	1/16	1/16	1/16	1/16	1/4
4	1/4	0	0	0	1/4
$P_X(x)$	1/2	1/4	1/8	1/8	

$$H(X) = -\sum_{x\in\mathcal{X}} p(x)\log p(x) = -\left[\tfrac{1}{2}\lg\tfrac{1}{2} + \tfrac{1}{4}\lg\tfrac{1}{4} + \tfrac{1}{8}\lg\tfrac{1}{8} + \tfrac{1}{8}\lg\tfrac{1}{8}\right]$$

$$= -\left[-\tfrac{1}{2} - \tfrac{1}{2} - \tfrac{3}{8} - \tfrac{3}{8}\right] = \tfrac{14}{8} = \tfrac{7}{4}$$

$$H(Y) = -\sum_{y\in\mathcal{Y}} p(y)\lg p(y) = -\left[\tfrac{4}{4}\lg\tfrac{1}{4}\right] = 2 \quad \checkmark$$

$$H(Y|X) = \sum_{x\in\mathcal{X}} p(x) H(Y|X=x) =$$

$$H(X|Y) = \sum_{y\in\mathcal{Y}} p(y) H(X|Y=y) = \tfrac{1}{4}H\left(\tfrac{1}{2},\tfrac{1}{4},\tfrac{1}{8},\tfrac{1}{8}\right)$$

$$+ \tfrac{1}{4}H\left(\tfrac{1}{4},\tfrac{1}{2},\tfrac{1}{8},\tfrac{1}{8}\right) + \tfrac{1}{4}H\left(\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4}\right) + \tfrac{1}{4}H(1,0,0,0)$$

$$H(X|Y) = \frac{1}{4}\left[-\frac{1}{2}\lg\left(\frac{1}{2}\right) - \frac{1}{4}\lg\left(\frac{1}{4}\right) - \frac{1}{8}\lg\left(\frac{1}{8}\right) - \frac{1}{8}\lg\left(\frac{1}{8}\right)\right]$$

$$+\frac{1}{4}\left[-\frac{1}{4}\lg\left(\frac{1}{4}\right) - \frac{1}{2}\lg\left(\frac{1}{2}\right) - \frac{1}{8}\lg\left(\frac{1}{8}\right) - \frac{1}{8}\lg\left(\frac{1}{8}\right)\right]$$

$$+\frac{1}{4}\left[-\frac{4}{4}\lg\left(\frac{1}{4}\right)\right] + 0$$

$$= \frac{1}{4}\left(\frac{7}{4}\right) + \frac{1}{4}\left(\frac{7}{4}\right) + \frac{1}{4}(2) = \frac{14}{16} + \frac{2}{4} = \frac{7}{8} + \frac{4}{8} = \frac{11}{8} \checkmark$$

$$H(Y|X) = \sum_{x \in X} p(x)\, H(Y|X=x)$$

$$= \frac{1}{2}H\left(\frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{2}\right) + \frac{1}{4}H\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}, 0\right) + \frac{1}{8}H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0\right) + \frac{1}{8}H\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0\right) \checkmark$$

$$= \frac{1}{2}\left[-\frac{1}{4}\lg\left(\frac{1}{4}\right) - \frac{1}{8}\lg\left(\frac{1}{8}\right) - \frac{1}{8}\lg\left(\frac{1}{8}\right) - \frac{1}{2}\lg\left(\frac{1}{2}\right)\right]$$

$$+ \frac{1}{4}\left[-\frac{1}{4}\lg\left(\frac{1}{4}\right) - \frac{1}{2}\lg\left(\frac{1}{2}\right) - \frac{1}{4}\lg\left(\frac{1}{4}\right)\right]$$

$$+ \frac{1}{8}\left[-\frac{1}{4}\lg\left(\frac{1}{4}\right) - \frac{1}{4}\lg\left(\frac{1}{4}\right) - \frac{1}{2}\lg\left(\frac{1}{2}\right)\right]$$

$$+ \frac{1}{8}\left[-\frac{1}{4}\lg\left(\frac{1}{4}\right) - \frac{1}{4}\lg\left(\frac{1}{4}\right) - \frac{1}{2}\lg\left(\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[+\frac{2}{4} + \frac{3}{8} + \frac{3}{8} + \frac{1}{2}\right] + \frac{1}{4}\left[+\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right] + \frac{3}{8}\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{18}{8}\right] + \frac{3}{8} + \frac{3}{8} = \frac{9}{8} + \frac{6}{8} = \frac{13}{8} \checkmark$$

$$H(X,Y) = -\sum p(x,y) \lg p(x,y) = -\left[\frac{1}{8}\lg\left(\frac{1}{8}\right) + \frac{1}{16}\lg\left(\frac{1}{16}\right) + \cdots \right.$$

$$-\frac{1}{4}\lg\left(\frac{1}{4}\right) = \frac{1}{2}$$

$$-\frac{1}{8}\lg\left(\frac{1}{8}\right) = \frac{3}{8}$$

$$-\frac{1}{16}\lg\left(\frac{1}{16}\right) = \frac{1}{4}$$

$$-\frac{1}{32}\lg\left(\frac{1}{32}\right) = \frac{5}{32}$$

$$H(X,Y) = \frac{3}{8} + \frac{1}{4} + 2\left(\frac{5}{32}\right) + \frac{1}{4} + \cdots$$

$$= 1\left(\frac{1}{2}\right) + 2\left(\frac{3}{8}\right) + 6\left(\frac{1}{4}\right) + 4\underbrace{\left(\frac{5}{32}\right)}_{\frac{5}{8}}$$

$$= \frac{16}{32} + \frac{24}{32} + \frac{48}{32}$$

$$= \frac{4}{8} + \frac{6}{8} + \frac{12}{8} + \frac{5}{8} = \frac{27}{8} \quad \checkmark$$

$$H(X) - H(X|Y) = \frac{7}{4} - \frac{11}{8} = \frac{3}{8}$$

$$H(Y) - H(Y|X) = 2 - \frac{13}{8} = \frac{3}{8} \quad \checkmark$$

Pg 19 Cover/Thomas

$$D(p\|q) = \sum_{x \in \mathcal{X}} p(x) \log\left(\frac{p(x)}{q(x)}\right)$$

$$= (1-r)\log\left(\frac{1-r}{1-s}\right) + r\log\left(\frac{r}{s}\right) \quad \checkmark$$

$$D(q\|p) = \sum_{x \in \mathcal{X}} q(x) \log\left(\frac{q(x)}{p(x)}\right) = (1-s)\log\left(\frac{1-s}{1-r}\right) + s\log\left(\frac{s}{r}\right) \quad -$$

$r = s$

$r = 1/2 \quad s = 1/4$

$$D(p\|q) = 1/2 \log\left(\frac{1/2}{3/4}\right) + \frac{1}{2}\log(2)$$

$$= \frac{1}{2}\log\left(\frac{2}{3}\right) + \frac{1}{2}\log(2) \quad -$$