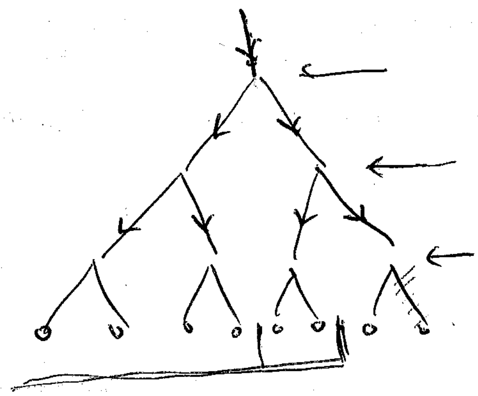
(8.1-1) We must be able to compare every input w/ every other
⇒ total order

(8.1-2) Random(a,b) has b-a+1 possible outputs

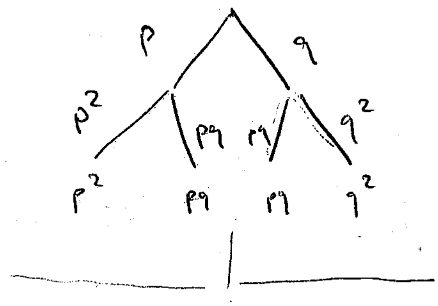


If b-a+1 is a power of 2, at each step i, a free flip a coin to determin the path to take

This uses routine Random(0,1). If b-a+1 is not a power of 2. Take the next largest power of 2 ∋ $2^N > b-a+1$. Then ... I was thinking one would add to many & then correct somehow.

Expected run $T(b-a+1) = \log_2(b-a+1) \cdot T(\text{Random}(0,1))$ for tree transversal.

5.1-3



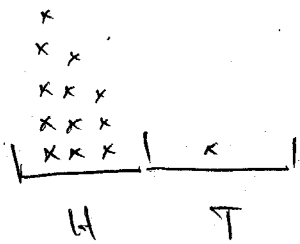
$p+q=1$

$(p+q)^n = 1$

$$\sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} = 1$$

$q(1-p)$

$p(p+q)$ $q(p+q)$



give coin w/ $p=1$ $q=0$.

$p=.9$ $q=.1$

$q + \Delta p$

~~let~~ p & q ~~w.o.l.o.g~~

& let $p > 1/2$ w.o.l.o.g. let $\Delta >$

$p - \Delta = \frac{1}{2}$ then $q + \Delta = \frac{1}{2}$

eq C.36

$E[X] = np$

X_i = ith flip comes up heads

Y_i = outcome of flip i

$X_i = I\{Y_i = H\}$ $\quad X = \sum_{i=1}^{n} X_i$

X = random variable = # of times we hire a new office assistant.

$E[X] = \sum_{x=1}^{n} x \Pr\{X = x\} = \sum x$ prob we hire x times.

X_i = indicator random variable stating whether we hire office assistant i, or not.

$E[X_i] = \Pr\{$candidate i is hired$\} = \Pr\{$candidate i is better than candidates $1, 2, \ldots, i-1\}$

$=$ (⊗ × ⊗ × × ⊗ × × ×)

n candidates sorted

pick i

$\frac{1}{i}$ why?

Because given i candidates what is prob you are looking at the best? $= \frac{1}{i}$ for only 1 is the best of

$$E[\overline{X}] = \sum_{i=1}^{n} \frac{1}{i} = \ln n + O(1)$$

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(5.2-1) hire exactly once ⇒ best candidate comes ~~first~~ 1st.

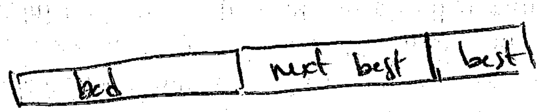
⇒ $\frac{1}{n}$

hire exactly n times ⇒ candidates are sorted worst to best before they arrive

$\frac{1}{n!}$ Because of all the combinations of candidate orders you get you must get the particularly special situation of sorted

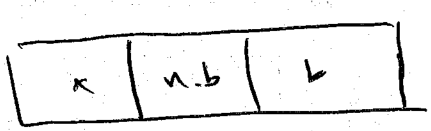
only 1 of the n! possible orders.

(5.2-2) $\frac{\cancel{2}}{n!}$ pick two candidates (best & next best) & order them next to



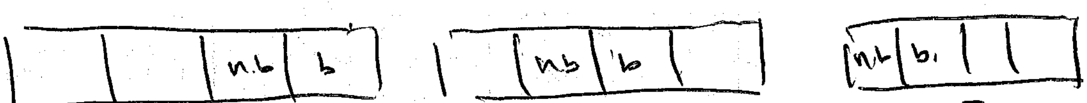
When I ~~ca~~ can move the pair next best & best up & down the list of n candidates.

n=3



choices = 2

n=4



choices = 3

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Choice for $n-1$

$\therefore$ Prob $= \frac{n-1}{n!}$

5.2-3

X_i = indicator random variable that die roll i comes

random variable $\{Y_i = 1\}$

$X_i^1 = I\{Y_i = 1\}$

$X_i^2 = I\{Y_i = 2\}$

$\vdots$

$X_i^6 = I\{Y_i = 6\}$

$E[X] = \sum_{x=n}^{6n} x\,p(x)$

Then $X = \sum_{i=1}^{n} \sum_{k=1}^{6} X_i^k$

$E[X] = \sum_{i=1}^{n} \sum_{k=1}^{6} E[X_i^k] = \frac{1}{6} \cdot 6n = n$? I don't think

this is correct.

5.2-4

$X_i =$ indicator random variable

$I\{\text{customer}_i \text{ gets his own hat}\}$

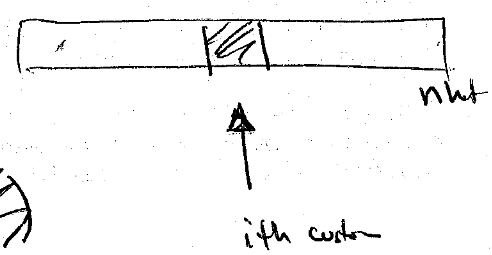
1, 2, 3, ..., n

$\underbrace{1, 2, 3, \ldots,}$ i, i+1, ..., n

$$X = \sum_{i=1}^{n} X_i$$

$$E[X] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} \frac{\binom{n}{n-1}}{n!} = \sum_{i=1}^{n} \frac{1}{(n-1)!}$$

$$= \frac{n}{(n-1)!}$$

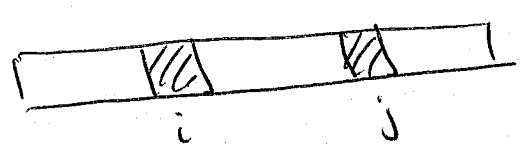


$\binom{n}{n-1}$

5.2-5

$X_{ij} =$ indicator random var to say we have an inversion between i & j

$E[X_{ij}] = ?$



$$X = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} X_{ij}$$

$$E[X_{ij}] = \frac{\{\binom{n}{2} \text{ but order does matter}\}}{n!}$$

$$E[X_{ij}] = \frac{2\binom{n}{2}}{n!} = \frac{\cancel{2}\,\frac{n!}{\cancel{2}!\,(n-2)!}}{\cancel{n!}} = \frac{1}{(n-2)!}$$

$$\therefore\ X = \frac{1}{(n-2)!}\sum_{i=1}^{n-1}(n-i-1+1) = \frac{1}{(n-2)!}\sum_{i=1}^{n-1} n-i \quad \ldots$$

5.3-1 ?

5.3-2 $A = \langle x_1, x_2, \dots, x_n \rangle$

Think so yes.

~~5.3-3~~ When $i=n$ the loop will not work

5.3-3 No we could just happen to switch two elements ... I would think no but I am not sure how to show this

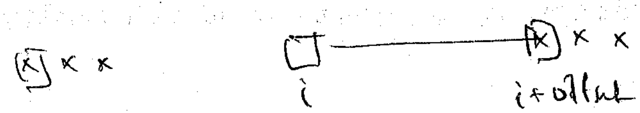
5.3-4 I think since one can

$n = 10$

offset $= 3$

$i=7$ dest $=10$

$i=8$ dest $=11$, dest $=1$ we wrap around



5.3-5

Prob 1st is unique $= 1$

Prob 2nd is unique $= \dfrac{n^3-1}{n^3}$

Prob 3rd is unique $= \dfrac{n^3-2}{n^3}$

$\vdots$

Prob ith is unique $= \dfrac{n^3-(i-1)}{n^3}$

$\vdots$

Prob nth is unique $= \dfrac{n^3-(n-1)}{n^3}$

Prob All are unique =

$$\Pr\{\ \cdot\ \} = 1 \cdot \left(\frac{n^3-1}{n^3}\right)\left(\frac{n^3-2}{n^3}\right)\cdots\left(\frac{n^3-(i-1)}{n^3}\right)\cdots\left(\frac{n^3-(n-1)}{n^3}\right)$$

$$= \left(1-\frac{1}{n^3}\right)\left(1-\frac{2}{n^3}\right)\cdots\left(1-\frac{i-1}{n^3}\right)\cdots\left(1-\frac{n-1}{n^3}\right)$$

$$= \prod_{k=0}^{n-1}\left(1-\frac{k}{n^3}\right)$$

$$\ln \Pr = \sum_{k=0}^{n-1} \ln\left(1-\frac{k}{n^3}\right) \underset{n \gg 1}{\approx} -\sum_{k=0}^{n-1}\frac{k}{n^3} = \frac{-1}{n^3}\sum_{k=0}^{n-1} k$$

$$\ln(1+x) = \ln(1) + \left.\frac{1}{1+x}\right|_{x=0} x + O(x^2)$$

$$= \frac{-1}{n^3}\sum_{k=1}^{n-1} k$$

$$= \frac{-1}{n^3}\left(\frac{(n-1)(n)}{2}\right)$$

$$= -\frac{(n-1)}{2n^2}.$$

$$\Pr \cong e^{-\frac{(n-1)}{2n^2}} = \exp\{$$

$$\approx 1 - \frac{n-1}{2n^2} = 1 - \frac{1}{2n} + \frac{1}{2n^2} \approx 1 - \frac{1}{2n}$$

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5.3-6 If two or more P[i]'s are identical permute these elements again using the same rule

$$B_k = \bigcap_{i=1}^{k} A_i \qquad B_k = A_k \cap B_{k-1}$$

$$\Pr\{B_k\} = \Pr\{A_k \cap B_{k-1}\} = \Pr\{B_{k-1}\}\Pr\{A_k \mid B_{k-1}\}$$

$$\Pr\{B_1\} = \Pr\{A_1\} = 1$$

$$\Pr\{A_k \mid B_{k-1}\} = \frac{n-k+1}{n} = \frac{n-(k-1)}{n}$$

$$\Pr\{B_k\} = \Pr\{B_{k-1}\} \cdot \Pr\{A_k \mid B_{k-1}\}$$

$$= \Pr\{B_{k-2}\}\Pr\{A_{k-1} \mid B_{k-2}\}\Pr\{A_k \mid B_{k-1}\}$$

$$= \cdots$$

$$= \Pr\{B_1\}\Pr\{A_2 \mid B_1\}\Pr\{A_3 \mid B_2\}\cdots\Pr\{A_{k-1} \mid B_{k-2}\}\Pr\{A_k \mid B_{k-1}\}$$

$$= 1\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\cdots\left(\frac{n-(k-1)}{n}\right)$$

$$= 1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{k-1}{n}\right)$$

$$\Pr\{B_k\} \le e^{-1/n}e^{-2/n}\cdots e^{-\frac{(k-1)}{n}} = \exp\left\{-\sum_{i=1}^{k-1}\frac{i}{n}\right\}$$

$$\Pr\{B_k\} \le \exp\left\{-\frac{k(k-1)}{2n}\right\} \le \frac{1}{2}$$

when $-\frac{k(k-1)}{2n} \le \ln(1/2)$ (*)

Prob all k birthdays are distinct is at most 1/2 when equality in * holds

$$k(k-1) \ge 2n\ln 2$$

$\Rightarrow$ $k^2 - k - 2n\ln 2 = 0$

$$k = \frac{+1 \pm \sqrt{1 + 4(2n\ln 2)}}{2}$$

$\therefore$ $k \ge \underbrace{\frac{(1 + \sqrt{1 + 8n\ln 2})}{2}}_{\underline{23}}$ $n = 365$

$X_{ij} = I\{$ person i & j have the

$$= \begin{cases} 1 & i_B = j_B \\ 0 & \text{oth} \end{cases}$$

$$E[X_{ij}] = \frac{1}{n}$$

X = # of simultaneous birth days

$$X = \cancel{\sum_{i=1}^{k}\sum_{j=1}^{k} X_{ij}} \quad \sum_{i=1}^{k}\sum_{j=i+1}^{k} X_{ij}$$

$E[X] =$

$$E[X] = \sum_{i=1}^{k} \frac{1}{n}(k-(i+1)+1) = \frac{1}{n}\sum_{i=1}^{k}(k-i) = \frac{k^2}{n} - \frac{1}{n}\sum_{i=1}^{k} i$$

$$= \frac{k^2}{n} - \frac{1}{n}\frac{k(k+1)}{2} = \frac{2k^2-k^2-k}{2n} = \frac{k^2-k}{2n} = \frac{k(k-1)}{2n} \checkmark$$

$$\therefore \quad k(k-1) > 2n$$

$$? \quad k^2 - k - 2n = 0 \qquad k = \frac{1 + \sqrt{1-8n}}{2}$$

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$$\frac{k(k-1)}{2n} \geq 2$$

$$k^2 - k - 4n \geq 0$$

$$k = \frac{+1 \pm \sqrt{1-16n}}{2}$$

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balls to toss to obtain success is given by geometric distribution

$E[X] = 1/p = \frac{1}{1/b} = b$

balls to toss ∀ bin to contain 1 ball.

if a ball falls in an empty bin a "hit".

expected # of tosses to obtain b hits.

$n = \sum_{i=1}^{b} n_i$ n_i is n_i = # tosses until a "success" of a bernoulli trial

→ geometric distribution.

$E[n_i] =$ ∀ toss in the ith stage then we have $b-(i-1)$ bins empty, thus prob of success = & $i-1$ bins full ∴

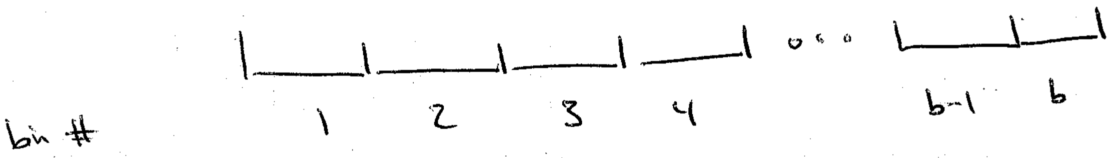
$\to \frac{1}{p}$

Stage # 1 1st toss $p = \frac{b-(i-1)}{b}$

Stage #2 all tosses after Stage #1 & until we hit an empty bin

Stage #3 " " " " 2 & " " " " " "

$\therefore E[n_i] = \frac{b}{b-(i-1)}$



$\#$ balls fell in a given bin b^* ? follows a ~~[illegible]~~ binomial

dist w/ prob of success $= 1/b$

$b(k; n, 1/b)$ w/ n balls tossed

$$b(n; n, 1/b) = \binom{n}{n} p^n q^{n-k} = 1(1/b)^n (1 - 1/b)^{n-1} \Big|_{k=n} =$$

$$= \frac{1}{b^n}\left(\frac{b-1}{b}\right)^{n-k}$$

$$= \left(\frac{1}{b}\right)^n$$

(C.36) $b(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$

$E[X] = \underline{np}$

= expected # of balls in a given bin.

$\therefore \; E[X] = \frac{n}{b}$

$$E[n] = \sum_{i=1}^{b} E[n_i]$$

$$= \sum_{i=1}^{b} \frac{b}{b-(i-1)} = b \sum_{i=1}^{b} \frac{1}{b-(i-1)}$$

$$= b \sum_{i=0}^{b-1} \frac{1}{b-i} = b\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{b}\right]$$

$$= b \sum_{i=1}^{b} \frac{1}{i} = b(\ln b + O(1))$$

A_{ik} = event that streak of length at least k begins w/ flip i.

= flips i, i+1, i+2, ..., ~~i+k-1~~ , i+k-1 of length k yield only heads.

$$i+k-1-i+1 = k$$

$1 \leq k \leq n$ k = streak length.

i = streak starting location

$1 \leq i \leq n-k+1$ $n-(n-k+1)+1 = k$

$$\Pr\{A_{ik}\} \leq \frac{1}{2^k}$$

why equal? Since the streaks can be longer than kth length.

Assume streaks can be only of length k

$$\Pr\{A_{ik}\} = \left(\frac{1}{2}\right)^k$$

$$k = \cancel{\ldots} \quad 2\lceil \lg n \rceil$$

$$\Pr\{A_{i2\lceil \lg n \rceil}\} = \left(\frac{1}{2}\right)^{2\lceil \lg n \rceil} = \left(\left(\frac{1}{2}\right)^{\lceil \lg n \rceil}\right)^2$$

$$\leq \left(\left(\frac{1}{2}\right)^{\lg n}\right)^2 = n^{-2} = \frac{1}{n^2}$$

Where can a string of length $2\lceil \lg n \rceil$ begin?

From position # 1 to

$$n - 2\lceil \lg n \rceil \underbrace{+1}_{?}$$ yes Add 1.

$$\cancel{\ldots} \quad n - (n - 2\lceil \lg n \rceil + 1) + 1 = 2\lceil \lg n \rceil$$

At most $n - 2\lceil \lg n \rceil + 1$ positions to begin such a streak.

$$\Pr\left\{ \bigcup_{i=1}^{n-2\lceil \lg n \rceil+1} \right\}$$

PS III CCR

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$$\Pr\{A_{i,2\lceil \lg n\rceil}\} \le \frac{1}{n^2}$$

$$\Pr\left\{\bigcup_{i=1}^{n-2\lceil \lg n\rceil+1} A_{i,2\lceil \lg n\rceil}\right\} \le \sum_{i=1}^{n-2\lceil \lg n\rceil+1} \Pr\{A_{i,2\lceil \lg n\rceil}\}$$

Boole's inequality:

$$< \sum_{i=1}^{n-2\lceil \lg n\rceil+1} 1/n^2 < \sum_{i=1}^{n} \frac{1}{n^2} < \frac{1}{n}$$

eq 5.10

$$E[L] = \sum_{j=0}^{n} j \Pr\{L_j\}$$

---

events L_j

$$E[L] = \sum_{j=0}^{2\lceil \lg n\rceil-1} j\Pr\{L_j\} + \sum_{j=2\lceil \lg n\rceil}^{n} j\Pr\{L_j\}$$

$$< \sum_{j=0}^{2\lceil \lg n\rceil-1} 2\lceil \lg n\rceil \Pr\{L_j\} + \sum_{j=2\lceil \lg n\rceil}^{n} n\Pr\{L_j\}$$

$$E[L] < 2\lceil \lg n \rceil 1 + n \cdot \sum_{j=2\lceil \lg n \rceil}^{n} \Pr\{L_j\}$$

$$= 2\lceil \lg n \rceil + n(1/n)$$

$$= 2\lceil \lg n \rceil + 1 = O(\lg n)$$

$n = 1000$ flips $\qquad 2\lceil \lg 1000 \rceil = 20$

$$\Pr\{A_{i,r\lceil \lg n \rceil}\} < \frac{1}{n^r}$$

$$\Pr\{\text{streak at least}\} < \frac{1}{(1000)^2}$$

$$\Pr\{A_{i,\lfloor \lg n/2 \rfloor}\} = \frac{1}{2^{\lfloor \lg n/2 \rfloor}}$$

$$\leq \left(1 - 1/\sqrt{n}\right)^{\frac{2n}{\lg n} - 1} < \left(e^{-\frac{1}{\sqrt{n}}}\right)^{\frac{2n}{\lg n} - 1}$$

$$= e^{-\left(\frac{2n}{\lg n} - 1\right)\frac{1}{\sqrt{n}}} =$$

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$$\frac{1}{\sqrt{n}}\left(\frac{2n}{\lg n} - 1\right) \geq \lg n$$

Can I express $\frac{1}{\sqrt{n}}\left(\frac{2n}{\lg n} - 1\right)$ in a polynomial

in $\lg n$? $= P_0 + P_1(\lg n) + P_2(\lg n)^2 + O((\lg n)^3)$

let $x = \lg n \Rightarrow n = e^x$

$\Leftrightarrow \frac{1}{e^{x/2}}\left(\frac{2e^x}{x} - 1\right)$ as a polynomial in x

$$\Rightarrow \frac{2}{x}e^{x/2} - e^{-x/2}$$

$$= \frac{2}{x}\sum_{k=0}^{+\infty}\frac{1}{k!}\left(\frac{x}{2}\right)^k - \sum_{k=0}^{+\infty}\frac{1}{k!}\left(\frac{-x}{2}\right)^k$$

$$= \sum_{k=0}^{+\infty}\frac{1}{k!}\left(\frac{2}{x}\left(\frac{x}{2}\right)^k - \left(\frac{-x}{2}\right)^k\right) = 1\left(\frac{2}{x}\cdot 1 - 1\right)$$

$$+ 1\left(\frac{2}{x}\left(\frac{x}{2}\right) + \frac{x}{2}\right)$$

$$+ \frac{1}{2}\left(\frac{2}{x}\left(\frac{x}{2}\right)^2 - \frac{x^2}{2^2}\right) + \dots$$

$$= \frac{2}{x} - 1 + 1 + \frac{x}{2} +$$

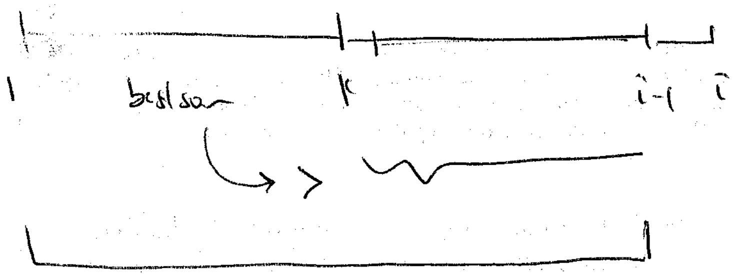
$$\geqslant$$

$$\Pr\{S\} = \sum_{i=k+1}^{n} \Pr\{S_i\}$$

~~$\Pr\{S_i\}$ = prob best applicant comes in position~~ i.

B_i = best candidate comes in at position i.

O_i =

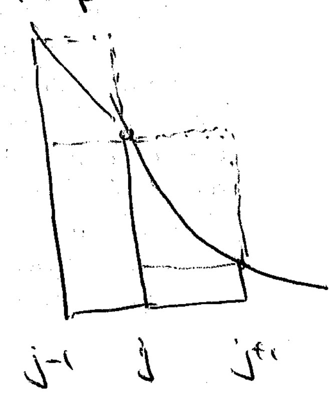


max value of all these pts must occur between $[1, k]$

$$\frac{1}{i-1-1+1} = \frac{k}{i-1}$$

$$\therefore \Pr\{S\} = \sum_{i=k+1}^{n} \frac{k}{n(i-1)} = \frac{k}{n} \sum_{i=k+1}^{n} \frac{1}{i-1} = \frac{k}{n} \sum_{i=k}^{n-1} \frac{1}{i}$$

$$\int \frac{dx}{x} \leq \sum_{i=k}^{n-1} \frac{1}{i} \leq \int_{k}^{n-2} \frac{dx}{x}$$



$$\frac{k}{n}(\ln n - \ln k)$$

$$= \frac{k}{n}\ln(n/k) = \frac{\ln(n/k)}{(n/k)}$$

$$\frac{1}{n}(\ln n - \ln k) + \frac{k}{n}\left(-\frac{1}{k}\right)$$

$$\frac{1}{n}(\ln n - \ln k - 1) = 0$$

$$\cancel{k=}\ \ln k = \cancel{\quad} \ln n - 1$$

$$= \ln(n/e)$$

$$k = n/e$$

w/ $k = n/e$.

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5.4-1 given n people, prob a person has a b-day on

(a) min is $\frac{1}{365}$

$\sqcup$

prob. after n attempts w/ $p = \frac{1}{365}$ we get a success is

$$q^{n-1} p$$

Thus getting a success denote a matching b-day.

~~The expected~~ #

$$q + q^2 + \cdots + q^n$$

Ask a ∞ # of people

$$\sum_{k=1}^{n} q^{k-1} p \geq \frac{1}{2}$$

If have n people ~~p~~ prob that one ~~~~ of them has the same b-day as a given day is $1 -$ Prob None have B-day x.

$\Rightarrow$ Prob None have B-day x $\Rightarrow$ fail n independent bernoulli trials

$\Rightarrow q^n$

$\therefore \; 1 - q^n \geq \frac{1}{2}$ w/ $q = 1 - \frac{1}{365} = .9972$

$\Rightarrow 1 - \frac{1}{2} \geq q^n$

so $q^n \leq \frac{1}{2} \Rightarrow n \ln q \leq \ln(1/2)$

$$n \geq \frac{\ln(1/2)}{\ln(q)} = 252.$$

(b) $p =$ prob 2 people have same b-day on specific date

$\left(\frac{1}{365}\right)^2$. "success"

Prob at least 2 people have given b-day

$\Rightarrow$ 1 − Prob No people have same b-day $=$ ~~[illegible])~~ ~~1−(?)q~~ $\frac{1}{2}$

07-05-02 ~~8~~3

$$1 - q^{\binom{n}{2}} > \frac{1}{2}$$

This is prob that we fail once $\forall$ pair of people in the room

$$-q^{\binom{n}{2}} > -\frac{1}{2}$$

$$q^{\binom{n}{2}} < \frac{1}{2}$$

$$\binom{n}{2} > \frac{\ln(1/2)}{\ln q}$$

$$\frac{1}{2} n(n-1) > \frac{\ln(1/2)}{\ln q}$$

$$n(n-1) > \frac{2\ln(1/2)}{\ln q}$$

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8.4.2 Let p = prob that two balls end up in the same bin

$p = \left(\frac{1}{b}\right)^2$ for every pair of ~~two~~ tosses

?

8.4.3 ?

8.4.4 ?

8.4.5 k-string = string of length k

over set $|S| = n$?

8.4.6 $E[B(0)] = ?$

$E[B(1)] = ?$

?

8.4.7 ?

(8-1) $n_0 = 0$ 000…0 ⇒ $n_0 = 0$

increment by 1 w/ prob. $\frac{1}{n_{i+1} - n_i}$

remain unchanged w/ prob. $1 - \frac{1}{n_{i+1} - n_i}$

If $n_i = i$ ordinary increment counter

Select $n_i = 2^{i-1}$

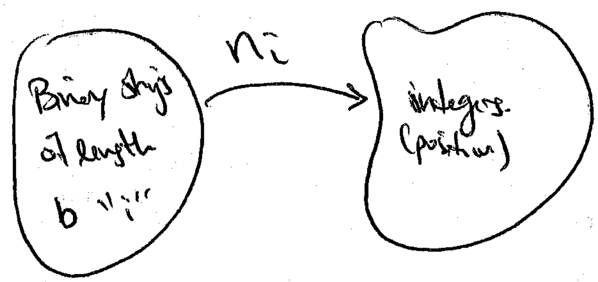
or $n_i = F_i$ ith Fibonacci #

(a) $E[n_k] = ?$ After k increment operations

$$n_{i+1} - n_i = 2^i - 2^{i-1} = 2^i(2-1) = 2^i$$

increment by 1 w/ prob $\frac{1}{2^i}$

$n_i = ?$



$n_1 = ?$ $n_1 = 1$ even if

$n_0 = 0$

Pick sequence n_i before hand. if $n_i = i$ then

Thus I know what value i represents $\forall i$

$= n_i$

2

07-03-02

But I can do several increment opperations w/out changing the value of the i (bit string)

(a) $E[n_n]$ = expected value of n_i after ~~#~~ n calls to increment.

=

$$n_k = \begin{cases} n_{k+1} & \text{w/ prob } \dfrac{1}{n_{k+1} - n_k} \\ n_k & \text{w/ prob } 1 - \dfrac{1}{n_{k+1} - n_k} \end{cases}$$

Call a "success" if we increment the counter from n_k to n_{k+1}

? How many success do we have after n trials ?

This is a binomal distribution w/ prob of success at the ith trial of $\underline{\underline{p_i}} = \dfrac{1}{n_{i+1} - n_i}$

~~E[N]~~ ~~#~~ $n_i = 2^i$ $\quad p_i = \dfrac{1}{2^i}$

$$\underbrace{|\_\_ p_1 \_\_|}_{N_1} \underbrace{\_\_ p_2 \_\_|}_{N_2} \underbrace{\_\_ p_3 \_\_|}_{N_3} \underbrace{\_\_ p_4 \_\_|}_{N_4} \cdots \underbrace{|\_\_ p_n \_\_|}_{N_n}$$

let N = Avg # of calls to increment to get to n_n

$\cancel{E[N]}$ $N = \sum_{i=1}^{n} N_i$

$\underline{E[N]} = \sum_{i=1}^{n} E[N_i] = \sum_{i=1}^{n} \frac{1}{p_i} = \sum_{i=1}^{n} (n_{i+1} - n_i) = \sum_{i=1}^{n} \Delta n_i$

$= n_i \Big|_1^{n+1} = n_{n+1} - n_1$

...

C = value of counter

$C = n_{i^*}$ some i^*

$E[C] = ?$

$= \cancel{E}i \; n_{E[i]} = n_{\#\text{ of successes}}$ ... ?

(b) $VAR[C] = n_{VAR[i]}$

(5-2)

(a)

```
listOfVisitedIndexes = {};
while(1) do
    i = random([1,n])
    if( i ∉ listOfVisitedIndexes)
        add i to listOfVisitedIndexes
        if( x == A[i]) return
    endif
    if ( length(listOfVisitedIndexes) == n) break
enddo
```

(b) These ~~are~~ are independent trials to get index i^*.

w/ $i^* \ni A[i^*] = x$.

$$p_{success} = \frac{1}{n}$$

$$E[\#\text{ of trials}] = \frac{1}{p} = n$$

(c) ~~Psuccess =~~ Again each trial is independent so

$$P_{success} = \frac{k}{n}$$

$$\therefore E[X] = \frac{1}{P_{succ}} = \frac{n}{k}$$

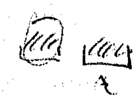
(d) Our algorithm terminates when we have filled all n bins w/ balls.

? How many throws do we need to do to obtain hits in all bins?

Thus if we call a success the placement of a ball in a new bin as the prob. that this happens changes as we throw. From i.e. problem is independent bernoulli trials in stages w/ probability p_i $\forall$ stage ...

Following results on pg 110 we get

$$E[X] = n(\ln(n))$$



(e) $$E[X] = \sum x p(x) = 1\left(\tfrac{1}{n}\right) + 2\left(\tfrac{1}{n}\right) + \cdots + n\left(\tfrac{1}{n}\right)$$

$$= \sum_{k=1}^{n} k\frac{1}{n} = \frac{1}{n}\sum_{k=1}^{n} k = \frac{1}{n}\frac{(n)(n+1)}{2}$$

$$= \frac{n+1}{2} = \underbrace{\Theta\left(\frac{n}{2}\right)}_{\text{expected running time}}$$

worst case running time is when element x is located as the last element of the input array. i.e. $E[X] = n$

(f) ~~B~~ Worst case would be when all of the elts are positioned at the right hand side of the array

All x values at k of them

$$E[X_{worst\ case}] = n-k$$

Average case

$$E[X_{Avg}] = 1\left(\frac{k}{n}\right) + \underbrace{2\left(\frac{n-k}{n}\right)\left(\frac{k}{n}\right) + 3\left(\frac{n-k}{n}\right)^2\left(\frac{k}{n}\right)}_{?}$$

(g) $E[X] = n$ in both cases

(h) Permuting the input array would simply duplicate the Random-search algo & should give results as before.

~~xxxx~~ except I don't think that technically a worst case ~~exi~~ analysis results since the randomization removes any bad behavior.

(ii) ?