5.5-3

We can prove that $G=(V,E)$ is a free tree by showing that $\forall$ two vertexes a ~~path~~ unique simple path exists. Pick $\{u,v\} \in V$.

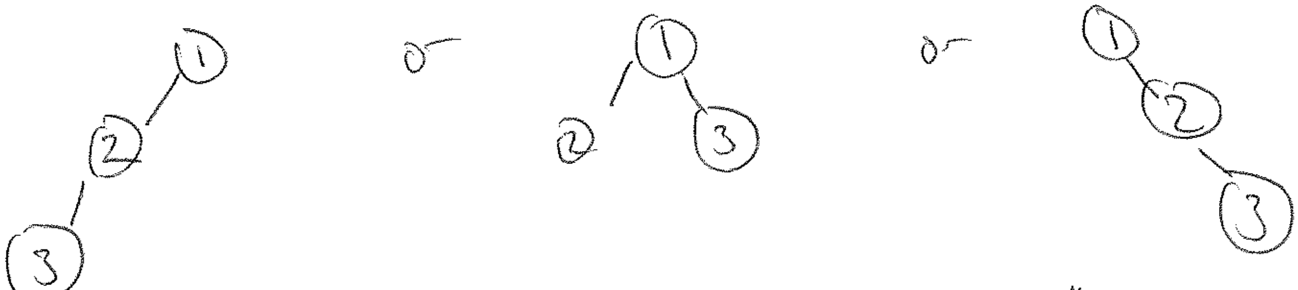
Then find the two paths from U to U_0 & V to U_0

Connecting these gives a path from U to V.

Since both U to U_0 & V to U_0 are distinct & unique the joined path is.

5.5-4 Consider a binary tree w/ 3 nodes

The 3 possible situations exist



# leaves :	1	2	1
# degree 2 nodes	0	1	0

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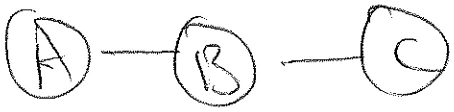
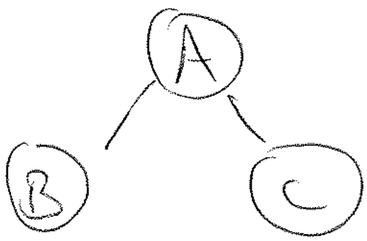
5.5-1



w/ permutations in the letters

w/ permutations in the letters

Ordered trees



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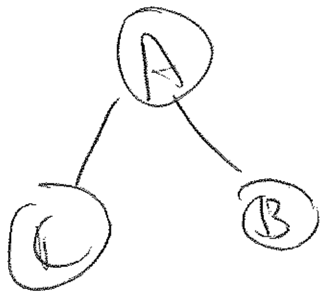
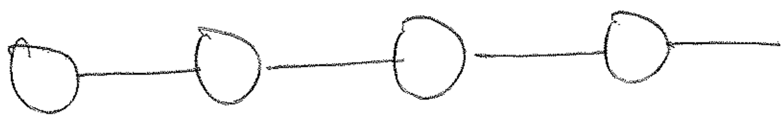
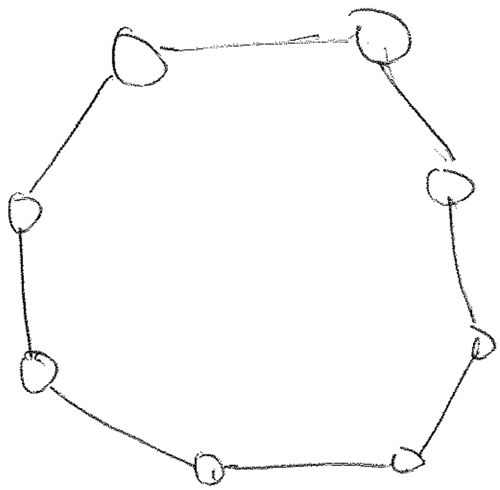
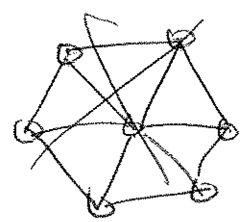


same w/ binary trees.

5.5-2







has cycle

Assume that for a ~~any~~ given binary tree of n nodes the # of degree two nodes equals # leaves − 1

To obtain a binary tree w/ n+1 nodes we must attach this final node to a leaf. Attaching to a leaf can either

1) not create a degree two node (attaching to the outside)

2) or create a degree two node

If in 1) # leaves does not change
+ # degree two nodes does not change
∴ # degree 2 nodes = # leaves − 1

In 2) # leaves increases by one
+ # degree two nodes increases by one but still
# degree 2 nodes = # leaves − 1.

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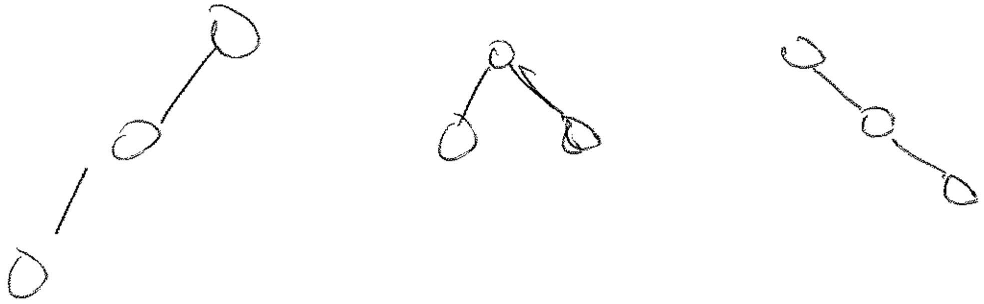
5.5-5 ~~The~~ ~~start~~ w/ 3 nodes 3 cases exist



height $\cancel{3}\ 2$ $\cancel{2}\ 1$ 2

height $\geq \lfloor \lg 3 \rfloor = 1$

Assume w/ n nodes

height $\geq \lfloor \lg n \rfloor$

Adding another node ~~con~~

if ; $\text{height}(n+1) = \text{height}(n) + 1 \geq \cancel{\lfloor \lg n+1 \rfloor}$

$\lfloor \lg n \rfloor + 1 \geq \lfloor \lg(n+1) \rfloor$

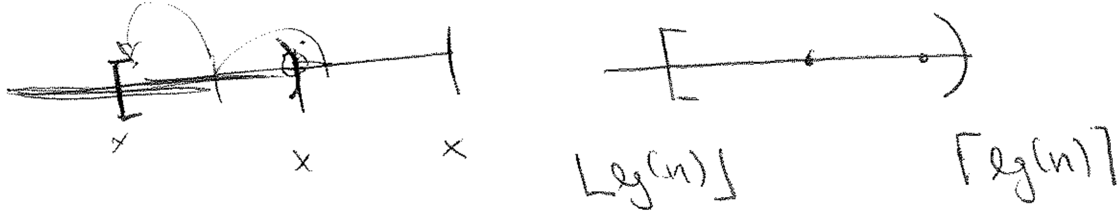
~~Since~~ ~~$x-1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x+1$~~

Since $\lg(n+1) = \cancel{\lg(n+1)}\ \lg\left(n\left(1+\frac{1}{n}\right)\right)$

$= \lg n + \lg\left(1+\frac{1}{n}\right)$

$\lg\left(1+\frac{1}{n}\right) < 1 \quad \forall n \geq$ "3" "2"

$$\lfloor \lg(n+1) \rfloor = \lfloor \lg(n) + \epsilon \rfloor$$



$$\leq \lfloor \lg(n) \rfloor + 1$$

Thus $\text{height}(n+1) \geq \lfloor \lg(n+1) \rfloor$

If Adding another node does <u>not</u> increase the height of the graph

$\text{height}(n+1) = \text{height}(n) \geq \lfloor \lg n \rfloor$

How show if does not increase the height of the graph?

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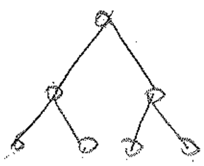
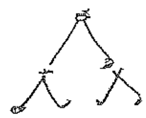
(5.5-6)

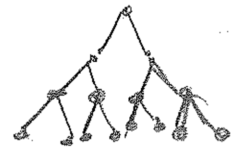
Full Binary tree w/ 3 = n internal nodes & k = 2 levels:



levels: k = 2



levels: k = 3



$i = 0 + 1 + 1 = 2$

$e = 2 + 2 + 2 + 2 = 8$

$i = 2$, $e = 8$, $n = 3$, $i + 2n = 8$ ✓

$i = 2 + 4(2) = 10$, $e = 8(3) = 24$, $n = 7$, $i + 2n = 10 + 14 = 24$ ✓

Check $e \overset{?}{=} i + 2n$

$8 = 2 + 2(3) = 8$ yes ✓

A full Binary tree w/ 3 internal nodes has 2 levels

# internal nodes	# levels	# leaves
3	2	$2^2 = 4$
$\sum_{k=0}^{2} 2^k = 2^0 + 2^1 + 2^2 = 7$	3	$2^3 = 8$
⋮	⋮	
$\sum_{p=0}^{k-1} 2^p \equiv n$	k	2^k

... By induction

$$\sum_{p=0}^{k-1} 2^p = 2^k - 1 = n$$

$$n = \sum_{p=0}^{k-1} 2^p = \left. \frac{2^p}{2-1} \right|_0^k = 2^k - 1$$

$\therefore \; n = 2^k - 1$

Now $e = (k) \cdot \#$ of leaves For a full binary tree w/ k internal levels

$= k \cdot 2^k$

$+ \quad i = (1)0 + 2(1) + 2^2(2) + \cdots 2^{k-1}(k-1)$

$i(2) = (1)0 + 2(1)$

$i(3) = i(2) + 4(2)$

$$\therefore \quad i(k) = \sum_{i=1}^{k} (i-1) 2^{i-1} = \sum_{i=0}^{k-1} i 2^i =$$

$$\left\{ \sum_{k=0}^{n-1} (a+kr) q^k = \frac{a - [a+(n-1)r] q^n}{1-q} + \frac{rq(1-q^{n-1})}{(1-q)^2} \right.$$

In my case $a = 0, \; r = 1, \; q = 2$

$\Rightarrow$ The

$$\sum_{k=0}^{n-1} kq^k = \frac{-(n-1)q^n}{1-q} + \frac{q(1-q^{n-1})}{(1-q)^2}$$ w/ $a=0$, $r=1$

$$\sum_{k=0}^{n-1} k2^k = -\frac{(n-1)2^n}{(-1)} + \frac{2(1-2^{n-1})}{1}$$

$$= (n-1)2^n + 2 - 2^n$$

$$= (n-2)2^n + 2$$

Thus $i(k) = (k-2)2^k + 2$

Thus $i(k) + 2n(k) = (k-2)2^k + 2 + 2(2^k - 1)$

$$= k2^k - \cancel{2^{k+1}} + \cancel{2^{k+1}} = k2^k$$

$$\stackrel{?}{=} e(k) = k2^k \quad \checkmark$$

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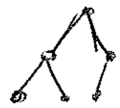
9 5.5-7



Ans

$$\sum_x w(x) = \sum_x 2^{-d} = 3(2^{-2}) = \frac{3}{4} < 1$$

A binary tree of depth 0 has 1 node

$$\sum_x w(x) = 2^{-0} = 1 \le 1$$ thm depth 0

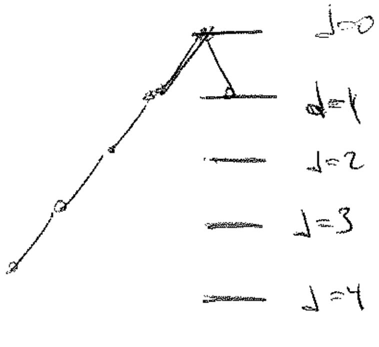
A binary tree w/ depth 1 can have

depth 1

$1(2^{-1})$, $1(2^{-1})$, or $2(2^{-1}) =$

$\frac{1}{2}$ $\frac{1}{2}$ 1 All less than 1.

Assume $\forall$ binary trees of depth d that $\sum_x 2^{-d} \le 1$



Now the largest sum of binary tree of depth d would be

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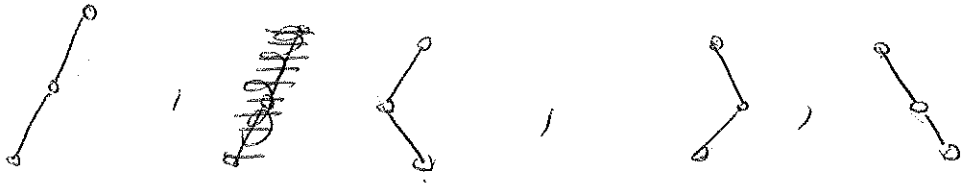
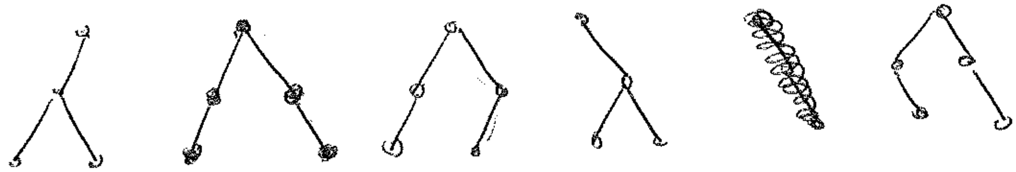


all have the same kraft sum.

of $(1)2^{-2} = \frac{1}{4}$



All have the same craft #.

$2\left(\frac{1}{2^2}\right) = \frac{1}{2}$

$\frac{1}{2} + \frac{1}{2} = 1$

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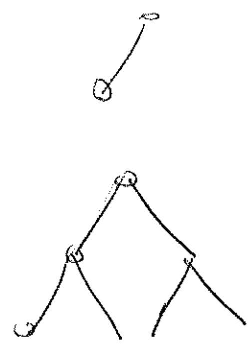
5.5-8 binary tree of depth $d=0$ has 0 leaves

depth $=1$ has 1, 2 leaves

depth $=2$ 1, 2, 3, 4 leaves

depth $=3$ has $1, 2, 3, \ldots, 2^d$



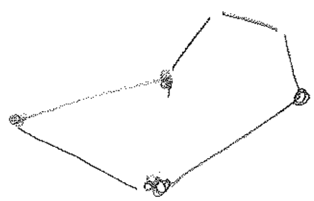
given L leaves I must ~~can~~ have a tree of depth

$\cancel{d_{max}} = \cancel{\lg_2 L}$ $\lfloor \lg_2 L \rfloor = d_{min}$.

$d_{Max} = +\infty$

5-1

(a) As a tree has the property that any two vertices are connected by a simple path, we can pick a base node call it v_0 ~~or all vertices adjacent to v_0~~ color it, and color all neighbors of it a different color.



This means that if we progressively color our vertices w/ alternating colors we don't have to worry about a conflicting color since no paths can exist.

(1) ⇒ (2)

(b) If G is bipartite ⇒ $\exists$ V_1 & V_2 ∋

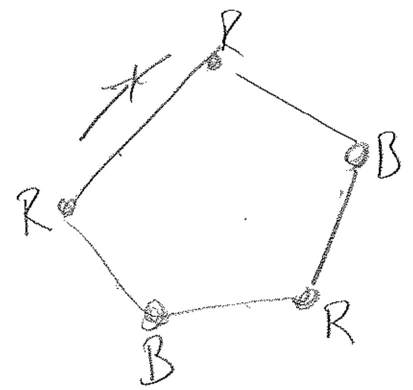
$G=(V,E)$ $V = V_1 \cup V_2$ ∋ $(u,v) \in E$ iff

$u \in V_1$ & $v \in V_2$ or $u \in V_2$ & $v \in V_1$. Now color all V_1 red & all V_2 black, This is a 2-coloring of the graph.

(2) ⇒ (3) Assume G did contain a cycle of odd length then G could not be 2 colorable for picking a vertex

05-~~06~~-02 2

From that cycle ~~[illegible]~~ and walking the cycle would result in a contradiction at the last step



(3) ⇒ (1) Pick a vertex v_0 assign it to a set V_1

then partition every vertex one away from v_0 into set V_2 ...

thus forming two sets V_1 & V_2. Let $(u,v) \in E$

I claim $u \in V_1$ & $v \in V_2$ or vice versa.

Assume BWOC that $u \in V_1$ and $v \in V_1$?

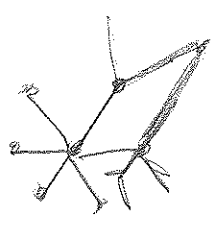
How show this induces a cycle of odd length?

(C) Find the maximum degree call it v_0.

Then color it color 0 w/ d other colors placed

on the adjacent vertices For every newly placed

colors go to ~~[illegible]~~ all the

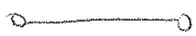


(5-1) (d)

G has $O(|V|)$ edges then G can be colored w/ $O(\sqrt{|V|})$ colors

(5-2)

(a) For any graph w/ $|V| \geq 2$ $\exists$ two vertices w/ the same degree.

If $|V| = 2$ 

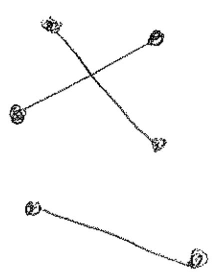
and each vertex has degree 1. 4

Assume for $|V| = k$ there exist two vertices w/ degree d.

?

(b) every graph w/ $|V| = 6$ ~~contains~~ has all nodes w/ degree 5

or



?

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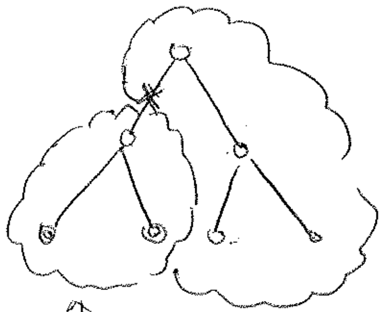
(5-3) Bisecting Trees

(a)



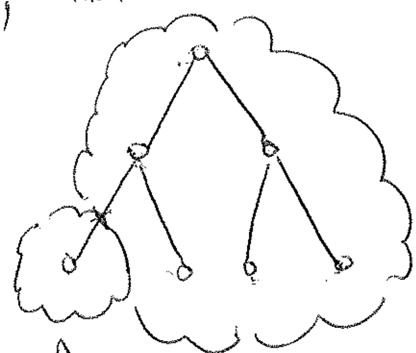
n = 7 vertex binary tree

$$\frac{3}{4}(7) = \frac{21}{4} = 5.25$$

$|A| = 3 \le \frac{3n}{4}$ ✓ $|B| = 4 < \frac{3}{4}n$ ✓

The largest set A or B would be formed from the removal of a leaf, ~~that~~ from a full tree

(b)



A B

set B has $6 > 5.25$!! elements ~~→~~ ?

(c) ?

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$$\frac{n-1}{k-1} \overset{?}{\geq} \frac{n}{k}$$

$$k(n-1) \overset{?}{\geq} n(k-1)$$

$$\cancel{kn} - k \overset{?}{\geq} \cancel{nk} - n$$

$$-k \geq -n \Rightarrow n \geq k \quad \text{Yes..}$$

$$\frac{n-2}{k-2} \overset{?}{\geq} \frac{n}{k}$$

$$\Downarrow$$

$$\cancel{kn} - 2k \geq \cancel{nk} - 2n \Rightarrow \quad n \geq k \quad \text{yes.}$$

$$\vdots$$

$$\frac{n-k+1}{1} \overset{?}{\geq} \frac{n}{k}$$

$$kn - k^2 + k \geq n \Rightarrow \cancel{k^2 - k - kn \geq -n}$$

$$n(k-1) - k(k-1) \overset{?}{\geq} 0$$

$$(k-1)(n-k) \geq 0 \quad \text{yes.}$$

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$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 2\cdot 1}$$

$$\le \frac{n^k}{k!} \le \frac{e^k n^k}{k^k} = \left(\frac{en}{k}\right)^k$$

$$k! \ge \left(\frac{k}{e}\right)^k \qquad \frac{1}{k!} \le \left(\frac{e}{k}\right)^k$$

$k = \lambda n$

$$\binom{n}{\lambda n} \le \frac{n^n}{(\lambda n)^{\lambda n}(n-\lambda n)^{n-\lambda n}} = \frac{n^n}{(\lambda n)^{\lambda n}((1-\lambda)n)^{(1-\lambda)n}}$$

$$= \left[\frac{n}{(\lambda n)^{\lambda}((1-\lambda)n)^{(1-\lambda)}}\right]^n = \left[\frac{\cancel{n}}{\lambda^{\lambda}(1-\lambda)^{(1-\lambda)}\cancel{n^{\lambda+1-\lambda}}}\right]^n$$

$$= \left[\left(\frac{1}{\lambda}\right)^{\lambda}\left(\frac{1}{1-\lambda}\right)^{1-\lambda}\right]^n$$

$$= 2^{nH(\lambda)}$$

$\rightarrow$ $\cancel{n}H(\lambda) = \cancel{n}\lg\left[\left(\frac{1}{\lambda}\right)^{\lambda}\left(\frac{1}{1-\lambda}\right)^{1-\lambda}\right]$ $\quad H(0) = 0$

$+ \quad H(1) = 0$

$$H(\lambda) = -\lambda\lg\lambda - (1-\lambda)\lg(1-\lambda)$$

6.1-1 [illegible]

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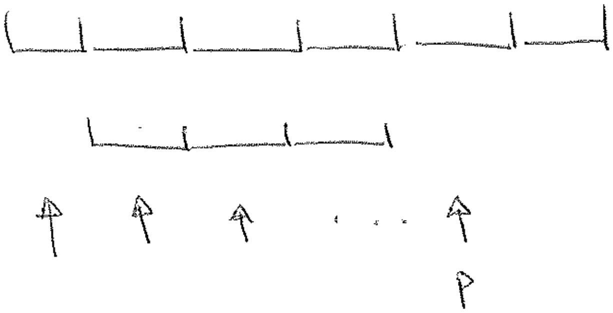


length n

(a)

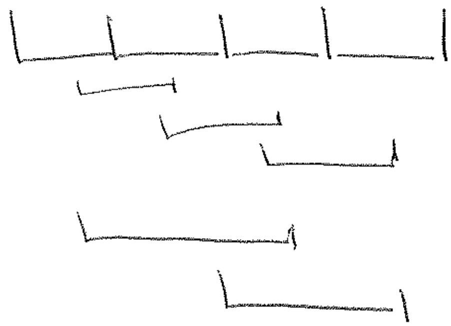
length k

p

$p + k = n \Rightarrow p = n - k$ Why is this way?

$\therefore$ $n-k+1$ substrings.

(b) n size 1 substrings



$n-1$ size two substrings

$n-2$ " 3 "

$\vdots$

$n-n+1$ size n substring

$$\therefore \sum_{k=1}^{n} n-k+1 = \sum_{k=1}^{n} n+1-k = (n+1)n - \sum_{k=1}^{n} k$$

$$= n(n+1) - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}$$

6.1-7

$$f(\{T,F\}^n) = \{T,F\}^1$$

There are 2n ~~choices~~ choices to determine the input argument

Then

x_1	x_2	$x_3 \cdots$	x_n	f
T	T	T $\cdots$	T	T, F
T	T	T $\cdots$	F	T, F

each different of the

2^n rows

$$\underbrace{2 \cdot 2 \cdots \cdot 2}_{\text{a product of 2 } \forall \text{ row}} = 2^{2^n} \text{ different functions}$$

$$f(\{T,F\}^n) = \{T,F\}^m$$

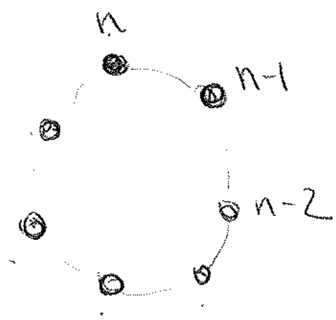
x_1	$x_2 \cdots$	x_n	f_1	$f_2 \cdots$	f_m
			T,F	T,F $\cdots$	T,F

2^m choices $\forall$ output

2^n rows

$$\underbrace{2^m \cdot 2^m \cdots 2^m}_{\#\text{ of rows}} = (2^m)^{2^n} = 2^{m \cdot 2^n}.$$

6.1-3



$n=7$

$n!$

6.1-4 $S = \{1, 2, \ldots, 100\}$ 3 #'s

∃ sum even in S & there are $1, 3, 5, 7, \ldots, 99$

$2, 4, 6, \ldots, 100$

$(2k+1)$ $\{k = 0, 1, \ldots, 49\}$ 50 odd #'s $\left\{ \frac{99-1}{2} = \frac{98}{2} = 45+4 = 49 \right.$

$2k$ $\{k = 1, 2, \ldots, 50\}$ 50 even #'s

To have an even sum I must pick

1 or: 2

3 even #s

1 even #
2 odd #s

$$\binom{50}{3} + \binom{50}{1}\binom{50}{2}$$

6.1-5

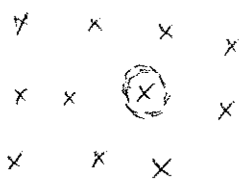
$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)!}{k(k-1)!(n-1-(k-1))!}$$

$$= \frac{n}{k}\binom{n-1}{k-1}$$

6.1-6

$$\binom{n}{k} = \frac{n!}{(n-k)!k!} = \frac{n(n-1)!}{(n-k)(n-k-1)!k!} = \frac{n}{n-k}\binom{n-1}{k}$$

6.1-7



$$\binom{n}{k} = \underbrace{\binom{n-1}{k}}_{\text{distinguished object is not chosen}} + \underbrace{\binom{n-1}{k-1}}_{\text{distinguished object is chosen}}$$

6.1-8

$\binom{0}{0} = 1$ $\binom{1}{0} = 1$

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{n-1}{k-1} \; \binom{4}{3} \; \binom{n-1}{k} \quad \binom{4}{4}$$

$\binom{5}{0}$ $\binom{5}{1}$ $\binom{5}{2}$ $\binom{5}{3}$ $\binom{5}{4}$ $\binom{5}{5}$

$\binom{6}{0}$ $\binom{6}{1}$ $\binom{6}{2}$ $\binom{6}{3}$ $\binom{6}{4}$ $\binom{6}{5}$ $\binom{6}{6}$

6.1-9

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\& \binom{n+1}{2} = \frac{(n+1)!}{(n+1-2)!\,2!} = \frac{(n+1)n\,\cancel{(n-1)}!}{\cancel{(n-1)}!\,2} = \frac{n(n+1)}{2} \checkmark$$

6.1-10

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)(n-2)\cdots 2\cdot 1}{k!\,(n-k)!} \qquad k \le \lceil \tfrac{n}{2} \rceil$$

$$= \frac{n(n-1)(n-2)\cdots(n-k+1)\,\cancel{(n-k)}!}{k!\,\cancel{(n-k)}!}$$

$$= \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}$$

$$\ln\binom{n}{k} = \sum_{p=n-k+1}^{n} \ln p - \sum_{p=1}^{k} \ln p$$

Because $\binom{n}{k} = \binom{n}{n-k}$ & $\binom{n}{k}$ is an increasing fn of

k

6.1-10

Since

$\binom{n}{k} = \binom{n}{n-k}$ $0 \le k \le n$ $n+1$

The values of $\binom{n}{k}$ repeats once k gets above

$\lceil \frac{n}{2} \rceil$

		1			n=0
	1		1		n=1
1		2		1	n=2
1	3		3	1	n=3
1	4	6	4	1	n=4

{ If n is even $\lceil \frac{n}{2} \rceil = \lfloor \frac{n}{2} \rfloor$ }

n odd $\lceil \frac{n}{2} \rceil - 1 = \lfloor \frac{n}{2} \rfloor$

Thus let n = odd

$\binom{n}{k}$ is unique $k = 0, 1, \ldots, \lfloor \frac{n}{2} \rfloor$

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

All one has to do is show $\binom{n}{k}$ is an increasing fn of k which is easy. $\binom{n}{k+1} > \binom{n}{k}$ & you are done

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(6.1-11)

$$\binom{n}{j+k} \le \binom{n}{j}\cdot\binom{n-j}{k}$$

$$\|$$

$$\frac{n!}{(j+k)!(n-j-k)!} = \frac{n!}{j!(n-j)!}\ \frac{j!(n-j)!}{(j+k)!(n-j-k)!}$$

$$= \binom{n}{j}\frac{(n-j)!}{k!(n-j-k)!}\ \frac{j!k!}{(j+k)!}$$

$$= \cancel{\binom{n}{j}\frac{(n-j)!}{k}}$$

j+k ~~terms~~ products

$$= \binom{n}{j}\binom{n-j}{k}\ \frac{(j(j-1)(j-2)\cdots 2\cdot 1)(k(k-1)\cdots 2\cdot 1)}{\underbrace{(j+k)(j+k-1)(j+k-2)\cdots 2\cdot 1}_{j+k\ \text{products}}}$$

W.L.O.G. Assume $j<k$

The

$$\frac{(j(j-1)(j-2)\cdots 2\cdot 1)(k(k-1)\cdots 2\cdot 1)}{(j+k)(j+k-1)(j+k-2)\cdots 2\cdot 1} = \frac{j!\cancel{k!}}{(k+j)(k+j-1)(k+j-2)\cdots(k+1)\cancel{k!}}$$

$$= \frac{j(j-1)(j-2)\cdots 2\cdot 1}{(j+k)(j+k-1)(j+k-2)\cdots \underset{(k+2)(k+1)}{\cancel{2\cdot 1}}}$$ 是算 < 1

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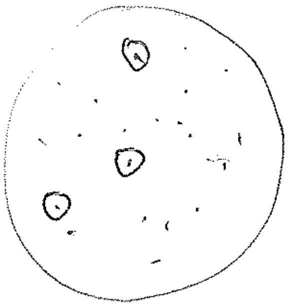
$\binom{n}{j+k} \le \binom{n}{j}\binom{n-j}{k}$ ~~TOTO~~. is,



given n items & choosing j+k is a smaller set than the # of ways to choose j items from n times # of ways to choose k items from the remaining n-j.

For an inequality req $\frac{j}{j+k} < 1$

$$\frac{1}{1+\frac{k}{j}} < 1 \Rightarrow \frac{k}{j} > 1$$

$$k > j$$

pick $n=4$ $k=3$ & $j=1$

$$\binom{4}{4} = 1 \overset{?}{<} \binom{4}{1}\binom{3}{3} = 4$$ yes.

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(6.1-12) ineq 6.10 eq 6.4

$$\binom{n}{k} \le \frac{n^n}{k^k (n-k)^{n-k}} \qquad \binom{n}{k} = \binom{n}{n-k}$$

$\forall \quad 0 \le k \le n.$

Assu $\quad \binom{n}{k} \le \dfrac{n^n}{k^k (n-k)^{n-k}} \quad \forall \quad 0 \le k \le k_1$

$k=0 \quad \binom{n}{k} = \cancel{n}\ 1 \le \dfrac{n^n}{1\, n^n} = 1 \ \checkmark$ w/ $0^0 = 1$

Th

$$\binom{n}{k_1+1} = \frac{n!}{(k_1+1)!\,(n-k_1-1)!} = \frac{n!\,(n-k_1)}{k_1!\,(k_1+1)\,(n-k_1)!} = \binom{n}{k_1}\left(\frac{n-k_1}{k_1+1}\right)$$

$$\therefore \quad \binom{n}{k_1+1} \le \frac{n^n}{k_1^{k_1}(n-k_1)^{n-k_1}} \cdot \left(\frac{n-k_1}{k_1+1}\right)$$

$$= \frac{n^n}{(k_1+1)^{k_1+1}(n-k_1)^{n-k_1-1}} \quad \frac{(k_1+1)^{k_1+1}}{k_1^{k_1}\,(k_1+1)}$$

$$= \frac{n^n}{(k_1+1)^{k_1+1}\,(n-k_1-1)^{n-k_1-1}} \quad \frac{(n-k_1-1)^{n-k_1-1}}{(n-k_1)^{n-k_1-1}} \cdot \frac{(k_1+1)^{k_1}}{k_1^{k_1}}$$

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$$\therefore \binom{n}{k_1+1} \le \frac{n^n}{(k_1+1)^{k_1+1}\,(n-(k_1+1))^{n-(k_1+1)}} \cdot$$

$$\left(1-\frac{1}{n-k_1}\right)^{n-k_1-1} \cdot \left(1+\frac{1}{k_1}\right)^{k_1}$$

Thm

Note: The extension to $k > n/2$ follows from 6.4 & the symmet in 6.10. i.e. If $k_1 > n/2$ $n-k_1 < n/2$

$$\binom{n}{k_1} = \binom{n}{n-k_1} \le \frac{n^n}{(n-k_1)^{n-k_1}\,(n-(n-k_1))^{n-(n-k_1)}}$$

$$= \frac{n^n}{k_1^{k_1}\,(n-k_1)^{n-k_1}} \qquad \text{QED}$$

This last term is

$$\left(\frac{n-(k_1+1)}{n-k_1}\right)^{n-(k_1+1)} \cdot \left[1+\frac{1}{k_1}\right]^{k_1}$$

6.1-13

$$\binom{2n}{n} = \frac{(2n)!}{n!\,n!}$$

Stirlings formula

$$n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n\left[1+\Theta\left(\frac{1}{n}\right)\right]$$

$$\therefore \quad (2n)! = \sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}\left[1+\Theta\left(\frac{1}{n}\right)\right]$$

$$n!^2 = 2\pi n\left(\frac{n}{e}\right)^{2n}\left[1+\Theta\left(\frac{1}{n}\right)\right]^2$$

$$= 2\pi n\left(\frac{n}{e}\right)^{2n}\left[1+\Theta\left(\frac{1}{n}\right)\right]$$

Thus

$$\binom{2n}{n} = \frac{\sqrt{4\pi n}\left(\frac{2n}{\cancel{e}}\right)^{2n}\left[1+\Theta(1/n)\right]}{\cancel{2}\pi n\left(\frac{n}{\cancel{e}}\right)^{2n}\left[1+\Theta(1/n)\right]}$$

$$= \frac{1}{\sqrt{\pi n}}\left(\frac{2n}{n}\right)^{2n}\frac{\left[1+\Theta(1/n)\right]}{\left[1+\Theta(1/n)\right]}$$

$$\binom{2n}{n} = \frac{2^{2n}}{\sqrt{\pi n}}\left(1+\Theta(1/n)\right)\left(1+\Theta(1/n)\right)$$

$$= \frac{2^{2n}}{\sqrt{\pi n}}\left(1+\Theta(1/n)\right)$$

6.1-14 $H(\lambda) = -\lambda \lg \lambda - (1-\lambda)\lg(1-\lambda)$

$\cancel{H'(\lambda)} =$

$$\lg x = \frac{\ln x}{\ln 2}$$

$$\lg' x = \frac{1}{x \ln 2}$$

$$H'(\lambda) = -\lg \lambda - \frac{\lambda}{\lambda \ln 2} + \lg(1-\lambda) - \frac{(\cancel{1-\lambda})}{(\cancel{1-\lambda})(-1)\ln 2}$$

$$= -\cancel{\lg \lambda + \lg} \ldots$$

$$= -\lg \lambda + \lg(1-\lambda) - \frac{1}{\ln 2} + \frac{1}{\ln 2} =$$

$$-\lg \lambda + \lg(1-\lambda) \underset{\text{set}}{=} 0$$

$$\lambda = 1-\lambda$$

$$2\lambda = 1$$

$$\lambda = 1/2 .$$

$$H''(\lambda) = -\frac{1}{\lambda \ln 2} - \frac{1}{(1-\lambda)\ln 2}$$

$$H''(1/2) \leq 0 \quad \therefore \text{max}$$

$$H(1/2) = 1/2 + \frac{1}{2} = 1$$

$$\Pr\{A|B\} = \frac{\Pr\{A\}\Pr\{B|A\}}{\Pr\{A\}\Pr\{B|}$$

$\Pr\{A|B\}$ $\qquad$ $\Pr\{\underset{\overset{\shortparallel}{A}}{\text{Biased coin choosen}} \mid \underset{\overset{\shortparallel}{B}}{\text{2 heads}}\} = ?$

~~$\Pr\{A\}\Pr\{A|B\}$~~

~~Pr~~ $P\{\text{Biased coin choosen}\} = \frac{1}{2}$

$P\{\text{2 heads} \mid \text{Biased coin choosen}\} = 1$

$P\{\text{Biased coin not choosen}\} = \frac{1}{2}$

$\Pr\{\text{2 heads} \mid \text{Biased coin w/ not choosen}\} = \frac{1}{4}$

$\Pr\{\text{Biased coin choosen} \mid \text{2 heads}\}$

$= P\{\text{Biased coin choosen}\}\cdot$

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6.2-1

$A_1 \cup A_2 \cup \cdots$ = decompose into disjoint sets

$= C_1 \cup C_2 \cup \cdots$

~~$C_1 = A_1 \setminus (A_2 \cup \cup$~~

$C_1 = A_1 \setminus (A_2 \cup A_3 \cup \cdots)$

$C_2 =$

$P\{A_1 \cup A_2 \cdots\} = \sum_i P\{C_i\}$

$P\{C_i\} < P\{A_i\}$

$P\{A_1 \cup \cdots\} \leq \sum_i P\{A_i\}$.

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6.2-2

R G
1 2

Sample space (r, g1, g2)

(H, H, H), (H, H, T), (H, T, H), (H, T, T),
yes
(T, H, H), (T, H, T), ~~(T, H, H)~~, (T, T, H), (T, T, T) = 8 total outcomes

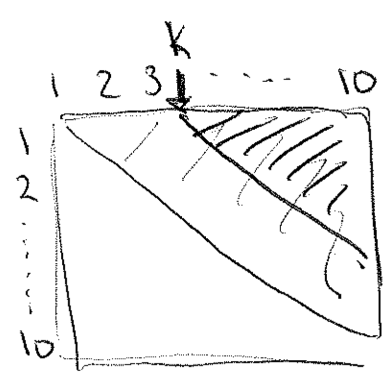
$\frac{1}{8}$

6.2-3

1 2 ... 10

1st choose w/ k,
have to get either k+1, k+2, ..., 10
next have k+2, k+3, ..., 10,

1 min



Vol in 3D space?

(6.2-4)

Possible outcomes ~~(x, y)~~ probability of such an event

H, H	p^2
H, T	$p(1-p)$
T, H	$p(1-p)$
T, T	$(1-p)^2$

$p(1-p) + p(1-p) = 2p(1-p)$

~~$p^2 + 2(p(1-p)) + (1-p)^2 = 1$~~

$$p^2 + (1-p)^2 = p^2 + 1 - 2p + p^2$$
$$= 1 - 2p + 2p^2$$
$$= 1 - 2p(1-p)$$

(6.2-5)

Possible outcomes w/ 2 flips ↓ prob of such event

H, H	1/4
H, T	1/4
T, H	1/4
T, T	1/4

$1/4 + 1/4 = 1/2$

$1/2$

(6.2-6)

$$P\{A|B\} + P\{\bar{A}|B\} = 1$$

$$P\{A|B\} = \frac{P\{A \cap B\}}{P\{B\}} \quad + \quad P\{\bar{A}|B\} = \frac{P\{\bar{A} \cap B\}}{P\{B\}}$$

Thus $P\{A|B\} + P\{\bar{A}|B\} = \dfrac{P\{A \cap B\}}{P\{B\}} + \dfrac{P\{\bar{A} \cap B\}}{P\{B\}}$

$$= \frac{P\{A\cap B\} + P\{\bar{A}\cap B\}}{P\{B\}}$$

since $A\cap B$ & $\bar{A}\cap B$ are ~~independent~~ mutually exclusive

$$= \frac{P\{A\cap B \cup \bar{A}\cap B\}}{P\{B\}} = \frac{P\{(A\cup\bar{A})\cap B\}}{P\{B\}}$$

$$= 1$$

6.2-7 Pr:

$$P\{A_1\cap A_2 \cdots \cap A_n\} = P\{A_1\}\cdot P\{A_2|A_1\} P\{ \cdots \} \cdot P\{A_n | A_1\cap A_2 \cdots A_{n-1}\}$$

This follows from

$$P\{A_1\cap A_2 \cap \cdots \cap A_n\}$$

$$= P\{A_1 | A_2 \cap A_3 \cap \cdots A_n\} \cdot P\{A_2\cap A_3 \cap \cdots A_n\}$$

Think of grouping sets as follows

$$A_1\cap A_2 \cap \cdots \cap A_n = (A_1\cap A_2\cap \cdots \cap A_{n-1})\cap A_n$$

The

$$P\{A_1 \cap \cdots A_n\} = P\{A_n | A_1 \cap \cdots \cap A_{n-1}\} \cdot P\{A_1 \cap \cdots \cap A_{n-1}\}$$

$$= P\{A_n | A_1 \cap \cdots \cap A_{n-1}\} \cdot$$
$$P\{A_{n-1} | A_1 \cap \cdots \cap A_{n-2}\} \cdot P\{A_1 \cap \cdots \cap A_{n-2}\}$$

$$= \cdots$$

$$= P\{A_n | A_1 \cap \cdots \cap A_{n-1}\} \cdot$$
$$P\{A_{n-1} | A_1 \cap \cdots \cap A_{n-2}\} \cdot P\{A_{n-2} | A_1 \cap \cdots A_{n-3}\}$$
$$\cdots P\{A_2 | A_1\} P\{A_1\}$$

6.2-8 ?

6.2-9 $Pr\{A \cap B | C\} = P\{A|C\} \cdot P\{B|C\}$

$P\{A \cap B\} \neq P\{A\} \cdot P\{B\}$ Not independent

A = flipping a coin Not an outcome of an event ?

B = obtaining a head

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6 2-10

$\underline{X}$ Y Z .

X $\underline{Y}$ Z .

X Y $\underline{Z}$.

$\boxed{\ }$ ⇒ this prisoner goes free.

$P\{X|Y\}$ = prob X go free given that Y is to die

guard tells X that Y is to be executed

$\underline{X}\ \underline{\underline{Y}}\ Z$, $\cancel{\underline{X}\ Y\ \underline{\underline{Z}}}$, $\cancel{X\ \underline{Y}\ \underline{\underline{Z}}}$, $X\ \underline{\underline{Y}}\ \underline{Z}$

$P\{X\} = \frac{2}{4} = \frac{1}{2}$

$P\{X\} = \frac{1}{2}$ since in one situation out of two X goes free.

6.2-10

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$

$P_{win} = \frac{1}{3}$ if no switch (i.e. original guessed chances)

P_{win} switch = ?

1	2	3
X		O
X	O	
O	X	
	X	O
O		X
	O	X

X = prize
O = shown

W.O.L.O.G Assume you originally pick curtain #1

Then P_{win} =

6.2-12

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$$\sum_{k=0}^{n} \cancel{\binom{n}{k}} (x+y)^k = (x+y)^n$$

Note: This problem may not be in the 1st edition

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

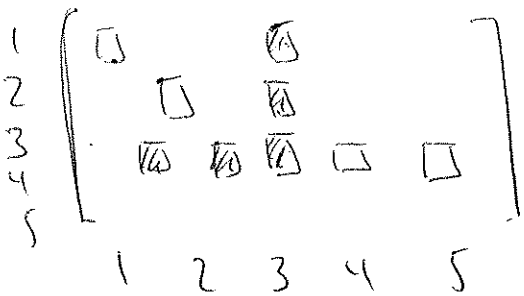
So $\frac{d}{dx} \rightarrow$

$$n(x+y)^{n-1} = \sum_{k=0}^{n} \binom{n}{k} k x^{k-1} y^{n-k}$$

Set $y = 1 = x$

$$n2^{n-1} = \sum_{k=0}^{n} \binom{n}{k} k$$

$\Pr\{X = 3\} = ?$ $\quad 5/36$



Def C.14 is $P\{A \mid B\} = \dfrac{\Pr\{A \cap B\}}{\Pr\{B\}}$

$$\Pr\{X = x \mid Y = y\} = \frac{\Pr\{X = x \text{ And } Y = y\}}{\Pr\{Y = y\}}$$

(H, H)
(H, T)
(T, H)
(T, T)

2 wins +3

$$E[X] = 6\Pr\{2\text{ heads}\} + 1\Pr\{1\text{ head } \& \text{ 1 tail}\} - 4\Pr\{2\text{ tails}\}$$

$=$

$$= 6\left(\tfrac{1}{4}\right) + 1\left(\tfrac{1}{2}\right) - 4\left(\tfrac{1}{4}\right)$$

$$= \frac{6 + 2 - 4}{4} = 1$$

$$E[X] = \sum_{i=0}^{\infty} i \Pr\{X = i\}$$

$$= \sum_{i=0}^{\infty} i\left[\Pr\{X \geq i\} - \Pr\{X \geq i+1\}\right]$$

$$= - \sum_{i=0}^{\infty} i\left[\Delta_i \Pr\{X \geq i\}\right]$$

$$= -i \Pr\{X \geq i\} \Big|_{0}^{\infty} + \sum_{i=1}^{\infty} \Pr\{X \geq i\}$$

$$0 + P\{X \geq 1\} - P\{X \geq 2\}$$

$$+ 2P\{X \geq 2\} - 2\Pr\{X \geq 3\}$$

$$+ 3P\{X \geq 3\} - 3P\{X \geq 4\}$$

$$\vdots$$

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$$= P\{X \geq 1\} + P\{X \geq 2\} + P\{X \geq 3\} + \cdots$$

$$\mathrm{Var}[X] = E[(X - E(X))^2]$$

$$= E[X^2 - 2X E(X) + E^2(X)]$$

$$= E[X^2] - 2E(X)^2 + E^2(X)$$

$$= E[X^2] - E^2(X) \qquad \text{eq C.26}$$

$$E(X^2) = \mathrm{Var}[X] + E^2(X)$$

6.3-1

$E[X] = ?$

X = sum of sum of two values?

2nd die

1st die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$E[X = \text{sum of values}]$

$$= 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) + 7\left(\frac{6}{36}\right)$$

$$+ 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + 10\left(\frac{3}{36}\right) + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right)$$

$E[X = \text{maximum of 2 values}] = ?$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

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$E[X = \text{maximum}]$

$$= 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right)$$

6.3-2

$E[X = \text{index of max elt of array}]$

$$= 1\left(\frac{1}{n}\right) + 2\left(\frac{1}{n}\right) + \cdots + n\left(\frac{1}{n}\right)$$

$$= \frac{\sum_{k=1}^{n} k}{n} = \frac{\frac{1}{2}n(n+1)}{n} = \frac{n+1}{2}$$

the sum for min.

6.3-3

$E[X = \text{payback from one play of carnival game}]$

$$= -1P\left\{\begin{array}{l}\text{players \# doesn't appear}\\ \text{on any dice}\end{array}\right\} + 1P\left\{\begin{array}{l}\text{his \# is on 1}\\ \text{dice}\end{array}\right\}$$

$$+ 2P\left\{\begin{array}{l}\text{his \# is on 2}\\ \text{dice}\end{array}\right\} + 3P\left\{\begin{array}{l}\text{his \# is on 3}\\ \text{dice}\end{array}\right\}$$

$6 \cdot 6 \cdot 6 = 36 \cdot 6 = 36 + 180 = 216$ possible outcomes of dice rolling.

$$P\left\{\begin{matrix}\text{his \# is on 3} \\ \text{dice}\end{matrix}\right\} = \frac{1}{216} = \left(\frac{1}{6}\right)^3$$

$$P\left\{\begin{matrix}\text{his \# is on 2} \\ \text{dice}\end{matrix}\right\} = \cancel{\frac{1}{216}} \left(\frac{1}{6}\right)^2$$

$$P\left\{\begin{matrix}\text{his \# is on 1} \\ \text{dice}\end{matrix}\right\} = \cancel{\frac{1}{216}} \left(\frac{1}{6}\right)$$

Not quite sure?

6.3-4 Not in 1st edition

$$E[\max(X,Y)] = \cancel{\sum_{X,Y} \max(X,Y) \Pr} = \sum_x \sum_y \max(x,y) \Pr\{X = x \text{ and } Y = y\}$$

$$\leq \sum_x \sum_y (x+y) \Pr\{X = x \text{ and } Y = y\}$$

$$= \sum_y \sum_x x \Pr\{X = x \text{ and } Y = y\} + \sum_x \sum_y y \Pr\{X = x \text{ and } Y = y\}$$

$$= \underbrace{\sum x \Pr\{X = x\}}_{E[X]} + \underbrace{\sum y \Pr\{Y = y\}}_{E[Y]}.$$

6.3-4

$$\Pr\{f(X) \cap g(Y)\} \stackrel{?}{=} \Pr\{f(X)\} \Pr\{g(Y)\}$$

How Do?

6.3-5 $\Pr\{X \geq t\} \leq \frac{E[X]}{t}$

$$\Longleftrightarrow \quad E[X] \geq t \Pr\{X \geq t\}$$

$$E[X] = \sum_x x \Pr\{X = x\} = \sum_{x<t} x \Pr\{X = x\} + \sum_{x \geq t} x \Pr\{X = x\}$$

$$\geq \sum_{x \geq t} x \Pr\{X = x\} \geq t \sum_{x \geq t} \Pr\{X = x\} = t \Pr\{X \geq t\}$$

6.3-6 $\bar{X}(s) \geq \bar{X}'(s)$

$$\Pr\{\bar{X} > t\} = \sum_{s \in S} \Pr\{\bar{X}(s) > t\}$$

$$= \sum_{\substack{s \in S \\ \ni \bar{X}'(s) \geq t}} \Pr\{\bar{X}(s) > t\} + \sum_{\substack{s \in S \\ \ni \bar{X}'(s) < t}} \Pr\{\bar{X}(s) > t\}$$

Since $\bar{X}'(s) \leq \bar{X}(s)$

$\nexists\, s \in S \ni \bar{X}'(s) > t \;\&\; \bar{X}(s) < t$

$$\geq \sum_{\substack{s \in S \\ \ni \bar{X}'(s) \geq t}} \Pr\{\bar{X}(s) > t\} = \sum_{s \in S} \Pr\{\bar{X}'(s) > t\}$$

6.3-8

$$E[\bar{X}^2] \; ? \; E[\bar{X}]^2$$

$$E[\bar{X}^2] = \mathrm{Var}[\bar{X}] + E^2[\bar{X}]$$

$$\Rightarrow \quad E[\bar{X}^2] \geq E^2[\bar{X}]$$

6.3-8 $X \in \{0,1\}$

~~Var[X]~~ The $E[X] = 0 \cdot p_0 + 1 p_1$

$$= p_1$$

$$E[X^2] = 0^2 p_0 + 1^2 p_1 = p_1$$

$$Var[X] = E[X^2] - E[X]^2$$

$$= p_1 - p_1^2 = p_1(1-p_1)$$

$$= E[X](1 - E[X]) = E[X](E[1-X])$$

6.3-~~10~~9

$$Var[aX] = E[a^2X^2 \cdot \;\; -$$

This follows directly from the linearity of $E[X]$

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$$E[X] = \sum_{k=1}^{\infty} k q^{k-1} p = \frac{p}{q} \sum_{k=0}^{\infty} k q^{k} \qquad \sum_{k=0}^{\infty} k q^{k} =$$

$$= \frac{p}{q}$$

— —

Roll 7 or 11 on sim

1st roll \ 2nd role	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P_7 = \frac{6}{36}$ $\qquad P_{11} = \frac{2}{36}$

$P_{7 or 11} = \frac{8}{36} = \frac{2}{9}$ = prob of success

$$E[X] = \frac{1}{p} = \frac{9}{2} = 4.5$$

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n trials how many succ. do I expect to get?

$$\Pr\{\underline{X}=k\} = \binom{n}{k} p^k q^{n-k}$$

k successes on n trials

trial #1 #2 #3 . . . . 0 0

p p p

$E[\underline{X}]$ = expectation of the # of successes I will get in n trials.

$$E[\underline{X}] = \sum_{k=0}^{n} k \Pr\{\underline{X}=k\} = \sum_{k=1}^{n} k \binom{n}{k} p^k q^{n-k}$$

$n-(k+1)$

$$= np \sum_{k=1}^{n} \binom{n-1}{k-1} p^{k-1} q^{n-k} = np \sum_{k=0}^{n-1} \binom{n-1}{k} p^{k} q^{n-k-1}$$

$$= np \sum_{k=0}^{n-1} \binom{n-1}{k} p^{k} q^{(n-1)-k} = np$$

$$E[\underline{X}_i] = p \cdot 1 + q \cdot 0 = p$$

eg ~~###~~ CLR
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$b(k;n,p) = \binom{n}{k} p^k q^{n-k}$ prob. we have k successes during n trials.

$= \cancel{\binom{n}{k}}$ eg C.6 is

$\leq \dfrac{n^n}{k^k (n-k)^{n-k}} p^k q^{n-k}$ $\binom{n}{k} \leq \dfrac{n^n}{k^k (n-k)^{n-k}}$

$= \left(\dfrac{n}{k}\right)^k \left(\dfrac{n}{n-k}\right)^{n-k} p^k q^{n-k}$

$= \left(\dfrac{np}{k}\right)^k \left(\dfrac{nq}{n-k}\right)^{n-k}$ ✓

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$$\frac{b(k;n,p)}{b(k-1;n,p)} = \frac{\binom{n}{k} p^k q^{n-k}}{\binom{n}{k-1} p^{k-1} q^{n-k+1}}$$

$$= \frac{\cancel{n!}}{k!(n-k)!} \cdot \frac{(k-1)!(n-k+1)!}{\cancel{n!}} \frac{p}{q} = \frac{1}{k}(n-k+1)\frac{p}{q}$$

$$= \frac{p(n-k+1)}{kq} = \frac{(1-q)(n-k+1)}{kq}$$

$$= \frac{p(n+1)}{kq} - \frac{k(1-q)}{kq} = \frac{p(n+1)}{kq} - \frac{k}{kq} + 1$$

$$= 1 + \frac{(n+1)p - k}{kq} \qquad \text{eq } C.40.$$

(6.4-1) geometric distribution p of success after k trials

$$pr\{X=k\} = q^{k-1} p$$

Check
$$pr\{X = \#\} = \sum_{k=1}^{\infty} q^{k-1} p = \frac{p}{q} \sum_{k=1}^{\infty} q^{k}$$

$$= \frac{p}{q}\left(\sum_{k=0}^{\infty} q^{k} - 1\right) = \frac{p}{q}\left(\frac{1}{1-q} - 1\right)$$

$q = 1-p$
$$= \frac{p}{q}\left(\frac{1}{p} - 1\right) = \frac{\cancel{p}}{q}\frac{(1-p)}{\cancel{p}} = 1 \checkmark$$

(6.4-2) Flip 6 fair coins before obtain 3 heads & 3 tails

$P_{success} = ?$ $\qquad P_{failure} = q = 1-p$

H,T H,T H,T H,T H,T H,T

$$\left(\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\right)$$

\# of possible out comes 2^6

\# of possible out comes that match our criteria $= \binom{6}{3}$

$$P_{success} = \frac{\binom{6}{3}}{2^6} = \frac{\frac{6!}{3!3!}}{2^6} = \frac{6!}{2^6 3!3!}$$

$$= \frac{\cancel{6}\cdot 5\cdot 4}{2^6 \; \cancel{3}\cdot 2} = \frac{20}{2^6} = \frac{5}{2^4}$$

$$E[X] = \frac{1}{p} = \frac{2^4}{5}$$

when X is ~~binom~~ geometrically distributed is

6.4-1 Axiom #2

$Pr\{S\} = 1$

$\Leftrightarrow \sum_{k=0}^{n} b(k;n,p) = 1$ $b(k;n,p) =$ k successes

$\Leftrightarrow \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = (p+1-p)^n = 1$

6.4-2 let (1) trial be flipping 6 coins

w/ success when we obtain <u>3 heads</u> + <u>3 tails</u>

$$P_{success} = \frac{\binom{6}{3}}{2^6} = \frac{\frac{6!}{3!^2}}{2^6} = \frac{6!}{3!^2 \cdot 2^6}$$

$$= \frac{\cancel{6} \cdot 5 \cdot 4}{\cancel{3} \cdot 2 \cdot 1} \frac{1}{2^6} = \frac{5 \cdot 4}{2^6} = \frac{20}{64} = .3125$$

Then # of trials until a "win" is given by a geometric distribution

$$E[X = succ] = \frac{1}{p} = 3.2$$

6.4-3

$$b(k;n,p) = \binom{n}{k} p^k q^{n-k}$$

$$= \cancel{\binom{n}{n-k}} \cdots \quad = \cancel{b(n-k;n+1}$$

$$= \binom{n}{n-k} q^{n-k} p^{n-(n-k)} \quad = b(n-k;n,q)$$

6.4-4

$$\max_{0 \le k \le n} b(k;n,p) =$$

Assuming $k=(n+1)p$ is an integer

$$\begin{cases} np-(1-p) < k < (n+1)p \\ (n+1)p-1 \le k \le (n+1)p \end{cases}$$

$$b(k;n,p)\Big|_{k_{max}}$$

$$= \binom{n}{k} p^k q^{n-k}$$

? How ?

$$\frac{n!}{(n-k)!\,k!} p^{(n+1)p} \cdot q^{n-(n+1)p}$$

$$= \frac{n!}{n-n\cdot}$$

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6.4-5

$Pr\{0 \text{ success}\} = b(0; n, p)$

$= \binom{n}{0} p^0 \cdot q^n = q^n = \left(1 - \frac{1}{n}\right)^n$

Recalling:

$\lim_{n \to +\infty} \left(1 + \frac{x}{n}\right)^n = e^x$

$\therefore \left(1 - \frac{1}{n}\right)^n \approx e^{-1}$.

$Pr\{1 \text{ success}\} = b(1; n, p)$

$= \binom{n}{1} p^1 q^{n-1} = n\left(\frac{1}{n}\right)\left(1 - \frac{1}{n}\right)^{n-1} = \left(1 - \frac{1}{n}\right)^{n-1} \approx e^{-1}$

6.4-6 Pr get same # heads

If Prof R gets k success ⇒ k heads $b(k; n, \frac{1}{2})$ = prob R gets k heads

If Prof G gets n-k success ⇒ k heads $b(n-k, n, \frac{1}{2})$ prob G gets k heads

Pr get same # heads $= \sum_{k=0}^{n} b(k; n, \frac{1}{2}) \cdot b(n-k; n, \frac{1}{2}) =$

$$= \sum_{k=0}^{n} b(k;n,1/2)^2$$

$$= \sum_{k=0}^{n} \binom{n}{k}^2 p^{2k} q^{2n-2k}$$

$$= \sum_{k=0}^{n} \binom{n}{k}^2 \left(\tfrac{1}{2}\right)^{2k} \left(\tfrac{1}{2}\right)^{2n-2k}$$

$$= \frac{1}{2^{2n}} \sum_{k=0}^{n} \binom{n}{k}^2 = ?$$

$$= \frac{1}{4^n} \sum_{k=0}^{n} \binom{n}{k}^2 = \frac{\quad}{2^n} \overset{?}{=}$$ How get $\binom{2n}{n}$?

6.4-7

$$b(k;n,\tfrac{1}{2}) = \binom{n}{k}\left(\tfrac{1}{2}\right)^n = \frac{\cancel{1}}{2^n}\ldots$$

$$H(x) = -x \lg x - (1-x)\lg(1-x)$$

$$= \frac{1}{2^n} \frac{n!}{(n-k)!\,k!} = \cancel{\frac{1}{2^n} \frac{(n-k+1)!}{(n-k)!}}$$

Wrong:

Assum $k \leq \frac{n}{2}$

$$\lg\left(\frac{(n-k+1)!}{(n-k)!}\right) = \lg(n-k+1)! - \lg(n-k)!$$

$$= \sum_{p=1}^{n-k+1} \lg p - \sum_{p=1}^{n-k} \lg(p)$$

$$\frac{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1}{}$$

$$\frac{n(n-1)(n-2)\cdots(n-k+1)\cancel{(n-k)}\cancel{(n-k-1)}\cdots\cancel{2}\cdot\cancel{1}}{k!\,\cancel{(n-k)}\cancel{(n-k-1)}\cancel{(n-k-2)}\cdots\cancel{2}\cdot\cancel{1}}$$

$$b(k;n,\tfrac{1}{2}) = \binom{n}{k}\frac{1}{2^n} = \frac{1}{2^n}\,\frac{(n-k+1)!}{k!}$$

$$\lg\left(\frac{(n-k+1)!}{k!}\right) = \sum_{p=1}^{n-k+1} \lg p - \sum_{p=1}^{k} \lg p$$

let $k \leq \frac{n}{2}$

$$=$$

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(6.4-8)

$$\Pr\{\underline{X} < k\} = \cancel{\Pr\{\underline{X} = k-1\}} + \Pr\{\underline{X} = k-2\}$$
$$+ \cdots + \cancel{\Pr\{\underline{X} = 1\}} + \underline{\Pr\{\underline{X} = 0\}}$$

$\le$

$b(k; n, p) = \binom{n}{k} p^k q^{n-k}$

$\boxed{b(n; n, p) = q}$

$$\Pr\{\underline{X} < k\} = \Pr\{\underline{X} = k-1\} + \Pr\{\underline{X} = k-2\}$$
$$+ \cdots + \Pr\{\underline{X} = 1\} + \Pr\{\underline{X} = 0\}$$

$$\Pr\{\underline{X} = 0\} = \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} 1 - p_i$$

$$\Pr\{\underline{X} = 1\} = \sum_{\substack{i=1 \\ i \neq k}}^{n} q_i + p_k = \sum_{\substack{i=1 \\ i \neq k}}^{n} 1 - p_i + p_k$$

$$\Pr\{\underline{X} = 2\} = \sum_{\substack{i=1 \\ i \neq k_1, k_2}}^{n} q_i + p_{k_1} + p_{k_2}$$

$\vdots$

$$\Pr\{\underline{X} = k-1\} = \sum_{\substack{i=1 \\ i \neq \{k_1, k_2, \cdots, k_{k-1}\}}}^{n} q_i + \sum_{i = \{k_1, k_2, \cdots, k_{k-1}\}} p_i$$

6.4-9

~~P~~ let $p > p_i$

$p' > p_i'$

~~Proof~~ From C.4-8 ~~$Pr\{X < k\} = 1$~~

$$Pr\{X \geq k\} = 1 - Pr\{X < k\}$$

Thus $Pr\{X \geq k\} \geq 1 - \sum_{i=0}^{k-1} b(i;n,p)$

$$Pr\{X' \geq k\} \geq 1 - \sum_{i=0}^{k-1} b(i;n,p')$$

So since $b(i;n,p) = \binom{n}{i} p^i q^{n-i} = \binom{n}{i} p^i (1-p)^{n-i}$

$$b(i;n,p') \geq b(i,n,p) \quad \forall \; i > \lfloor \tfrac{n}{2} \rfloor$$

$$b(i,n,p') \leq b(i,n,p) \quad \forall \; i \leq \lfloor \tfrac{n}{2} \rfloor$$

$$\Pr\{X-\mu \geq r\} \leq \left(\frac{\mu e}{r}\right)^r$$

$$\Pr\{X-\mu \geq r\} = \Pr$$

Markov's inequality:

$$\Pr\{X \geq t\} \leq \frac{E[X]}{t}$$

C.23: $E[X_1 X_2 \cdots X_n] = E[X_1]E[X_2]\cdots E[X_n]$

if X_i are mutually independent.

$$E\left[e^{\alpha(X_i - p_i)}\right] = p_i e^{\alpha q_i} + q_i e^{-\alpha p_i} \leq p_i e^{\alpha} + 1$$

$$\leq \exp\{p_i e^{\alpha}\}$$

$$E\left[e^{\alpha(X-\mu)}\right] \leq \prod_{i=1}^{n} \exp\{p_i e^{\alpha}\}$$

$$= \exp\left\{e^{\alpha}\sum_{i=1}^{n} p_i\right\} = \exp\{\mu e^{\alpha}\}$$

$$\Pr\{X-\mu \geq r\} \leq e^{-\alpha r}\exp\{\mu e^{\alpha}\}$$

$$= \exp\{\mu e^{\alpha} - \alpha r\}$$

let $\alpha = \ln(r/\mu)$

$$\Pr\{X - \mu \geq r\} \leq \exp\{r - \cancel{\alpha}\, r \ln(r/\mu)\}$$

$$= \frac{e^r}{e^{r\ln(r/\mu)}} = \frac{e^r}{(r/\mu)^r}$$

$$= \left(\frac{\mu e}{r}\right)^r$$

$$\Pr\{X - np \geq r\} = \sum_{k=\lceil np+r \rceil}^{n} b(k;n,p) \leq \left(\frac{npe}{r}\right)^r$$

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6.5-1

$\Pr\{X \leq 0\}$

let X = # of heads obtained on n flips of a coin.

$= q^n = (\tfrac{1}{2})^n$

V.S. $\Pr\{X < n\} < \dfrac{nq}{4np - n} b(n; 4n, p)$

$= \dfrac{n\tfrac{1}{2}}{n} b(n; 4n, \tfrac{1}{2})$

$= \tfrac{1}{2} b(n; 4n, \tfrac{1}{2}) = \tfrac{1}{2}\binom{4n}{n}(\tfrac{1}{2})^k(\tfrac{1}{2})^{n-k}$

$= \tfrac{1}{2}\binom{4n}{n}(\tfrac{1}{2})^n$

$\therefore \Pr\{X < n\} < (\tfrac{1}{2})^{n+1} \dfrac{4n!}{3n!\,n!} = (\tfrac{1}{2})^{n+1} \dfrac{4n(4n-1)(4n-2)\cdots(3n+1)}{n!}$

compare these two #'s

C-5-3 $\Pr\{X > k\} = \sum_{i=k+1}^{n} b(\cancel{\ }\, i; n, p)$ Not in 1st edition.

From

$b(i; n, p) = b(n-i; n, q)$ problem C.4-3

$$\Pr\{X > k\} = \sum_{i=k+1}^{n} b(n-i, n, q)$$

let $l = n - i$ $\quad l = \begin{cases} n-(k+1) & \text{lower band} \\ 0 & \text{upper band} \end{cases}$

$$= \sum_{l=n-(k+1)}^{0} b(l; n, q) = \sum_{l=0}^{n-(k+1)} b(l; n, q) = \tilde{\Pr}\{X < n-(k+1)\}$$

$$= \qquad = \sum_{l=0}^{n-k-1} b(l; n, q) = \hat{\Pr}\{X < n-k\}$$

$\cancel{\ } < \dfrac{(n-k)p}{nq-(n-k)} b($

$$\frac{(n-k)p}{nq-(n-k)} b(n-k \cancel{\ }; n, q)$$

$$\therefore \Pr\{\underline{X} > k\} < \frac{(n-k)p}{k-(1-q)n} b(n-(n-k\cancel{q});n,p)$$

$$\Pr\{\underline{X} > k\} < \frac{(n-k)p}{k-pn} b(k\cancel{q};n,p)$$

Corollary C.6 ✓

For Corollary C.7

$$\Pr\{\underline{X} > k\} < \frac{(n-k)p}{k-np} b(k;n,p)$$ By Corollary C.6

Not sure I follow . . .

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6.5-3

$$\sum_{i=0}^{k-1} \binom{n}{i} a^i = \cancel{\sum_{i=0}^{n} \binom{n}{i} a^i + \sum}$$

$\Pr\{X < k\}$ for a ~~bern~~ bernoulli process w/

$$b(k; n, p) = \binom{n}{k} p^k q^{n-k}$$

consider:

$$1 - \frac{a}{a+1} = \frac{1}{a+1}$$

Thus if $p = \frac{a}{a+1} < q$ ✓ $\quad q = \frac{1}{a+1}$

$$p^k q^{n-k} = \left(\frac{a}{a+1}\right)^k \left(\frac{1}{a+1}\right)^{n-k} = (a+1)^{-n} a^k$$

$$\Rightarrow a^k = \cancel{\frac{1}{(a+1)}} (a+1)^n p^k q^{n-k}$$

Thus

$$\sum_{i=0}^{k-1} \binom{n}{i} a^i = \sum_{i=0}^{k-1} (a+1)^n \binom{n}{i} p^i q^{n-i}$$

$$\rightarrow \sum_{i=0}^{k-1} \binom{n}{i} a^i = (a+1)^n \sum_{i=0}^{k-1} \binom{n}{i} p^i q^{n-i}$$

$$= (a+1)^n \sum_{i=0}^{k-1} b\left(i; n; \frac{a}{a+1}\right)$$

$$\leq (a+1)^n \frac{k\left(\frac{1}{a+1}\right)}{\left(n\left(\frac{a}{a+1}\right) - k\right)} b\left(k; n, \frac{a}{a+1}\right)$$

$$= (1+a)^n \frac{k}{(na - k(a+1))} b\left(k; n, \frac{a}{a+1}\right) \quad \checkmark$$

6.5-31

$$P_{\underline{u}}: \sum_{i=0}^{k-1} p^i q^{n-i}$$

know $$\sum_{i=0}^{k-1} b(i;n,p) = \sum_{i=0}^{k-1} \binom{n}{i} p^i q^{n-i}$$

Note:

$$\sum_{i=0}^{k-1} p^i q^{n-i} = q^n \sum_{i=0}^{k-1} p^i q^{-i} = q^n \sum_{i=0}^{k-1} \left(\frac{p}{q}\right)^i = q^n \cdot \frac{(p/q)^k - (p/q)^0}{p/q - 1}$$

$$\cancel{\sum_{k=k_1}^{k=k_2} r^k \neq \frac{r^{k+1}}{1-r}}$$

$$\Delta r^k = r^{k+1} - r^k = r^k(r-1)$$

$$\Rightarrow \sum_{k=k_1}^{k_2} r^k = \frac{1}{r-1} \sum_{k=k_1}^{k_2} \Delta r^k = \frac{1}{r-1} r^k \Big|_{k_1}^{k_2+1}$$

$$= \frac{r^{k_2+1} - r^{k_1}}{r-1}$$

Thus

$$\sum_{i=0}^{k-1} p^i q^{n-i} = q^n \left[\frac{(p/q)^k - 1}{p/q - 1} \right]$$

Thus

$$\sum_{i=0}^{k-1} p^i q^{n-i} = \quad \ldots \qquad \text{Not sure ?}$$

C.5-4

$$\Pr\{\mu - X \geq r\} \leq \left(\frac{(n-\mu)e}{r}\right)^r \quad (*)$$

Then CB is

$$\Pr\{X - \mu \geq r\} \leq \left(\frac{\mu e}{r}\right)^r$$

If eq (*) is true then for a sequence of bernoulli trials w/ all the same prob $\mu = np$ & then

$$\Pr\{np - X \geq r\} \leq \left(\frac{nqe}{r}\right)^r$$

I am not sure how to translate from CB to the requested prob although the

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C.5-8

$$E\left[e^{\alpha(X_i - p_i)}\right] = p_i e^{\alpha q_i} + q_i e^{-\alpha p_i}$$

dropping i's

$$p e^{\alpha q} + q e^{-\alpha p} \overset{?}{\leq} e^{\alpha^2/2}$$

$$\cancel{e^{(\alpha(p+q))^2}} =$$

$$p e^{\alpha(1-p)} + (1-p) e^{-\alpha p} \equiv F(p)$$

$$F'(p) = e^{\alpha(1-p)} + p e^{\alpha(1-p)}(-\alpha) + -e^{-\alpha p} + (1-p) e^{-\alpha p}(-\alpha)$$

$$\overset{\text{set}}{\equiv} 0$$

$$= e^{\alpha(1-p)}\left[1 - \alpha p\right] + e^{-\alpha p}\left[-1 - \alpha(1-p)\right] = 0$$

$$= e^{\alpha(1-p)}(1 - \alpha p) + e^{-\alpha p}(-1 - \alpha + \alpha p) = 0$$

$$= e^{\alpha(1-p)}(1 - \alpha p) - e^{-\alpha p}(1 + \alpha - \alpha p) = 0$$

$$e^{\alpha^2} = e^{(\alpha(p+q))^2} = \exp\left\{\alpha^2 (p^2 + 2pq + q^2)\right\} \quad ?$$

6.5-6

C.45 is $\Pr\{\bar{X} - \mu \geq r\} \leq \exp(\mu e^{\alpha} - \alpha r) \equiv F(\alpha)$

minimizing F w.r.t. α

$$\frac{dF}{d\alpha} = \exp(\mu e^{\alpha} - \alpha r)(\mu e^{\alpha} - r) \overset{\text{set}}{=} 0 \qquad (*)$$

$$\frac{d^2F}{d\alpha^2} = \exp(\mu e^{\alpha} - \alpha r)(\mu e^{\alpha} - r)^2 + \exp(\mu e^{\alpha} - \alpha r)(\mu e^{\alpha})$$

$$= \exp(\mu e^{\alpha} - \alpha r)\left[\mu^2 e^{2\alpha} - 2\mu e^{\alpha} r + r^2 + \mu e^{\alpha}\right]$$

$$= \exp(\mu e^{\alpha} - \alpha r)\left[\mu^2 e^{2\alpha} - (2r-1)\mu e^{\alpha} + r^2\right]$$

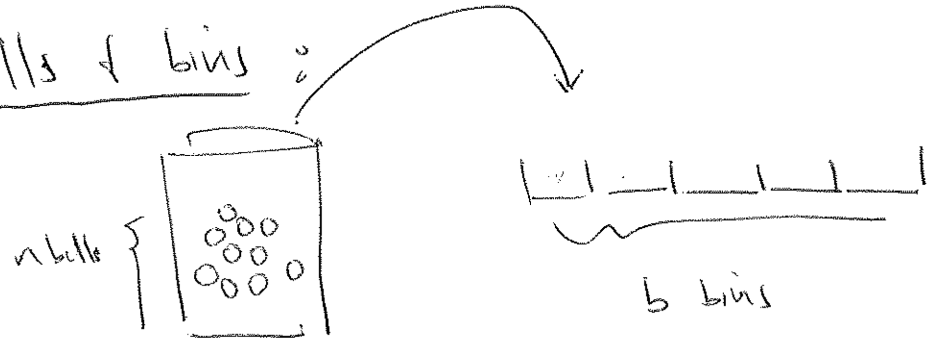
Solving eq $(*)$ gives $\alpha = \ln(r/\mu)$

in $$\frac{d^2F(\ln(r/\mu))}{d\alpha^2} = \exp(\cdots)\left[\mu^2\left(\frac{r^2}{\mu^2}\right) - (2r-1)r + r^2\right]$$

$$= \exp(\cdots)\left[\cancel{r^2} - \cancel{2r^2} + r + \cancel{r^2}\right] > 0$$

$\therefore$ $\alpha = \ln(r/\mu)$ is a <u>minimum</u>.

(6-1) Balls & bins:



(a) I have b choices for the 1st location of ball #1 (distinguishable balls $\Rightarrow$ I can label them)

$\Rightarrow b^n$

(b) | | |

2 bins $\Rightarrow$ 3 sticks

| ○ | ○ | ○ |

3 bins $\Rightarrow$ 4 sticks

to place n ball in b bins $\Rightarrow$ placing n balls & $b-1$ sticks

Then $\Rightarrow$ $n+b-1$ objects to place

$\Rightarrow (n+b-1)!$ ways to place these distinguishable objects w/ $(b-1)!$ repeated ways by permuting the sticks $\Rightarrow \dfrac{(n+b-1)!}{(b-1)!}$

(c) From pt b now the ordering of the balls does not matter

$$\therefore \frac{\frac{(n+b-1)!}{(b-1)!}}{n!} = \frac{(n+b-1)!}{(b-1)!\,n!} = \binom{n+b-1}{n}$$

(d) Balls identical + no bin may contain more

Assume $b > n$

$\Rightarrow$ ~~B_i~~ ~~A_i~~

$b(b-1)(b-2)\cdots(b-n+1)$ places to put balls

$\div$ by permutation of the balls

$$\Rightarrow \frac{b(b-1)(b-2)\cdots(b-n+1)}{n!} = \binom{b}{n}$$

(e) Balls identical $\Rightarrow$ compare

~~$b < n$~~ ~~$b(b-1)$~~...

place n balls between b-1 ~~B~~ sticks

$\cancel{\frac{(n+b-1)!}{n!\,(b!)}}$ doesn't enforce full bin requirement

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(e) want. $b(b-1)(b-2)\cdots(b-n+1)$ $n<b$

$(\underline{b(b-1)\cdots 2\cdot 1})$ $n>b$

? Not sure?

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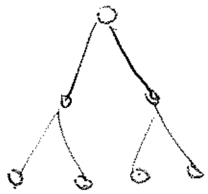
7 6.1-1

A full binary tree of height h has

$\cancel{\sum}$ elements



h=1 ; # = 2+1 = 3

h=2 ; # = 4+2+1 =

$$\sum_{i=0}^{h} 2^i = \frac{2^i}{2-1}\Bigg|_0^{h+1}$$

$$= 2^{h+1} - 1 \quad = \text{max \# of elements in heap}$$

& removing $(2^h - 1)$ elements (all but 1)

give min # of elements $= 2^{h+1} - 1 - 2^h + 1$

$$= 2^h(2-1) = 2^h.$$

6.1-2 Since a n element heap $\Rightarrow$ ~~no~~ n = # elements in heap

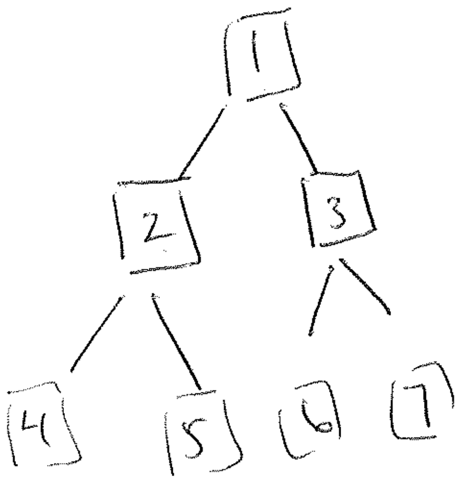
$$2^h \leq n \leq 2^{h+1} - 1 \underset{\neq}{<} 2^{h+1}$$

$\therefore$ $h \leq \lg n < h+1$ so $h = \lfloor \lg n \rfloor$

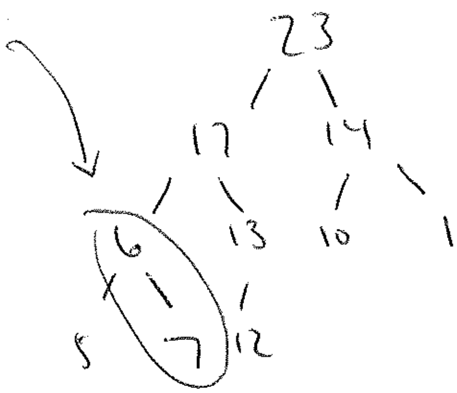
6.1-3 This follows directly from the max-heap property

6.1-4 ~~lower left corner of the binary tree~~ On any leaf.

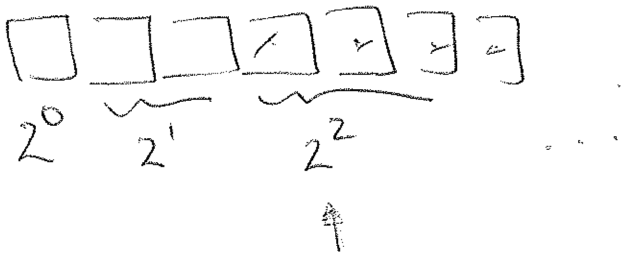
6.1-5 Yes, sorting least to largest 1 2 3 4



6.1-6 No is not a max-heap



6.1-7 The leaves are located at locations



$$n = \sum_{k=0}^{h-1} 2^k + p$$

$$n = (2^h - 1) + p$$

Problem not found in 1st edition

?

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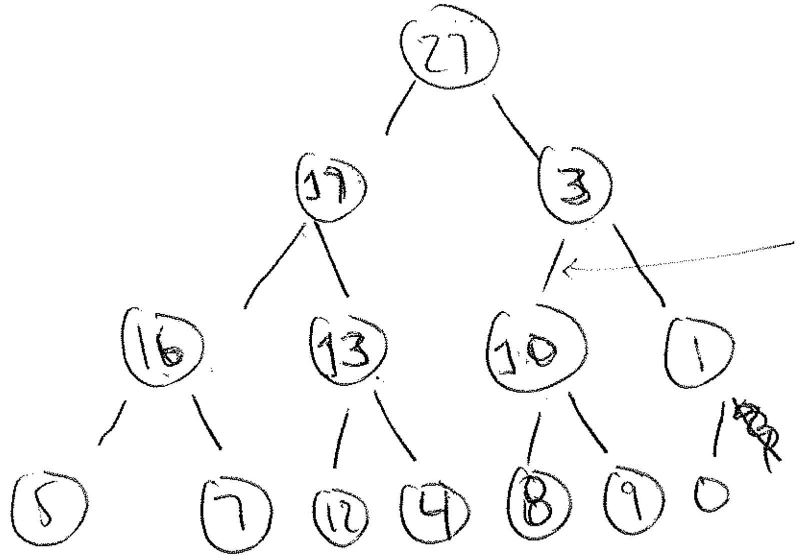
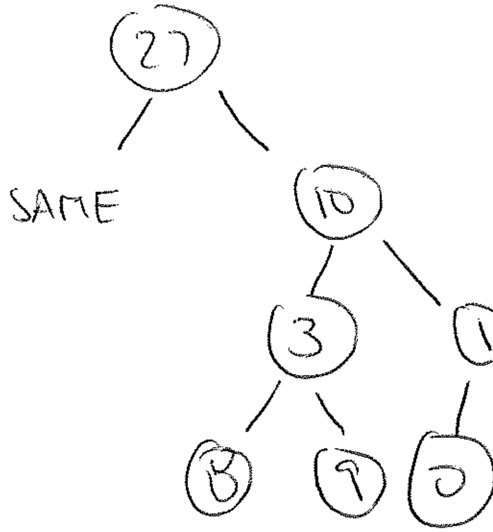
6.2-1

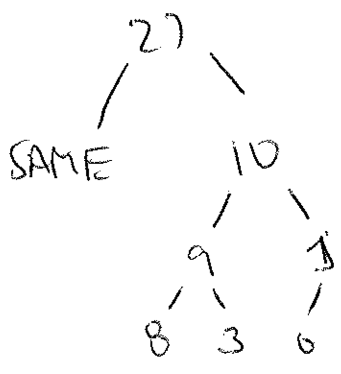


does not possess the max-heap property

Calling Max-Heapify(A,3)



call Max-Heapify(A,6)

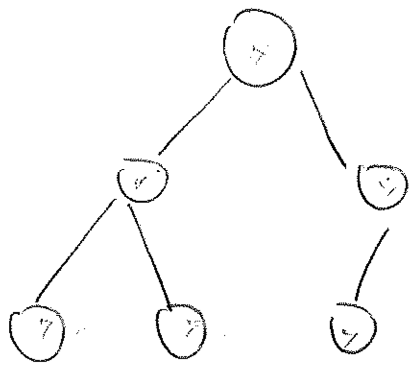


$T(n) \le T(2n/3) + \Theta(1)$

~~for~~ $O(n=10)$ sized tree

$\frac{2n}{3}$ comes from prob B.5-7 which says that every binary tree w/ L leaves contains a subtree w/ $\frac{L}{3}$ + $\frac{2L}{3}$ leaves inclusive

$n=6$



$n=3$

$$T(n) = T(2n/3) + \Theta(1)$$

$$= \Theta(1) + T(2n/3)$$

$$= \Theta(1) + \Theta(1) + T((\tfrac{2}{3})^2 n)$$

$$= 3\Theta(1) + T((\tfrac{2}{3})^3 n)$$

Stops when

$$(\tfrac{2}{3})^p n = 1$$

$$T(n) = \tfrac{2}{3}\lg n + T(1)$$

$$n = (\tfrac{3}{2})^p$$

$$\Rightarrow T(n) = O(\lg n) - O(n)$$

$$\Rightarrow p = \tfrac{2}{3}\lg n$$

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(6.2-2) This problem is not found in the 1st editio

The running time of min-Heapify should be identical to those of max-heapify.

Pseudocode for min-heapify would be

```
Min-Heapify(A, i)
    l ← left(i)
    r ← right(i)
    if( l ≤ heap-size(A) & A[l] ≤ A[i] )
        smallest ← l
    else
        smallest ← i
    if ( r ≤ heap-size(A) & A[r] < A[smallest] )
        smallest ← r.
    if ( smallest ≠ i ){
        exchange element i & smallest
        /* swap A[i] & A[smallest] */
        Min-Heapify( A, smallest )
    }
```

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(6.2-2) Max-heapify returns w/ no recursive calls & no actions

(6.2-3) When i > heap-size(A)/2 you are looking at nodes & there are no children thus the calls to left(i) & right(i) generate results ≥ heap-size(A)

(6.2-4) Do for I = i to heap-size(A)/2 do   I, largest

l ← left(I)
r ← right(I)
if l ≤ heap-size(A) & A(l) > A(I)
then largest ← l
else largest ← I
if (r ≤ heap-size(A) & A(r) > A(I))
largest ← r
if (largest ≠ I)
exchange (A[I] & A[largest])

enddo

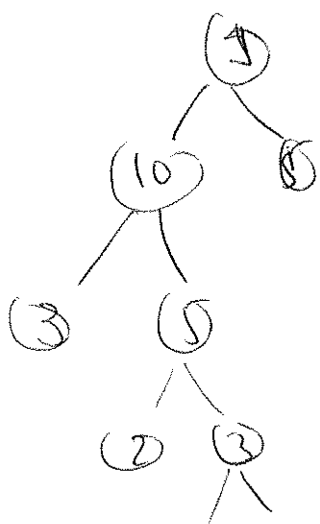
```
loop-Q = true;

while (loop-Q)
    ~~loop-Q = false~~;
    l ← left(i)
    r ← right(i)
    if (l ≤ Heap-size(A) & A[l] ≥ A[i])
        largest = l ;   ~~loop-Q = true~~;
    else
        largest = i
    endif
    if (r ≤ Heap-size(A) & A[r] ≥ A[i])
        largest = r;  ~~[illegible]~~
    endif
    if (largest ≠ i)
        ~~loop-Q = true~~
        exchange (A[i] & A[largest])
        loop-Q = true
        i = largest
    ~~endif~~ else
~~end while~~. loop-Q = false
```

end if

end while

7.2-5 Worst case is when the node i is less than every element on the path ~~of~~ in the tree from node i to the leaves going along maximum elements.



Then $T(n) = \Theta(1) + T(\frac{2}{3}n)$

$\Rightarrow T(n) = \Theta(\lg n)$

So for a general Max-heapify routine

$T(n) = \underline{\Omega}(\lg n)$

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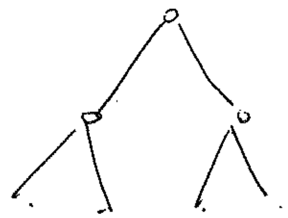
$n = 10 \qquad \left\lfloor \frac{n}{2} \right\rfloor + 1 = 6$



$n = 3$

$h = 1$ most $\left\lceil \frac{3}{4} \right\rceil = 1$

~~and~~ 2 nodes

$h = 2$, $n = 7$ nodes $\left\lceil \frac{7}{2^3} \right\rceil$

$$\sum_{h=0}^{\lfloor \lg n \rfloor} \left\lceil \frac{n}{2^{h+1}} \right\rceil O(h) = O\left(n \sum_{h=0}^{\lfloor \lg n \rfloor} \frac{h}{2^h} \right)$$

$=$

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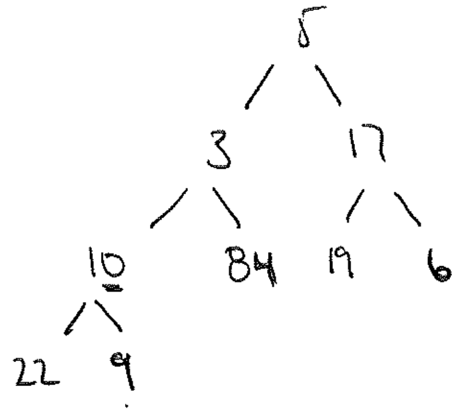
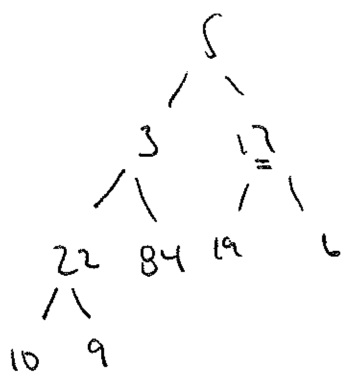
Ex. 3-1

$n = 9$

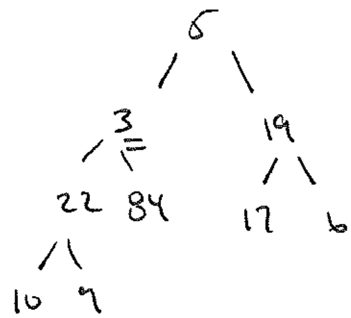


$\lfloor \frac{n}{2} \rfloor + 1 = \lfloor 4.5 \rfloor + 1 = 5$

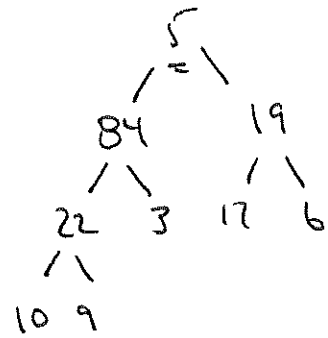
for i = 4 downto 1 do



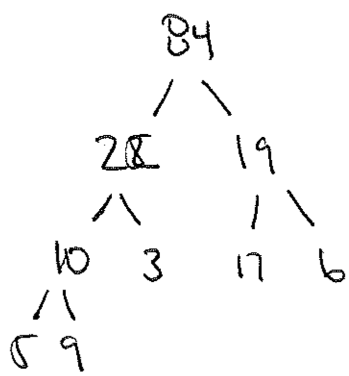
⇒



⇒



⇒



done

(Ex. 3-2) Because calling Max-Heapify(A, 1) will not work since left + right sub trees are not gaurunteed to be heaps

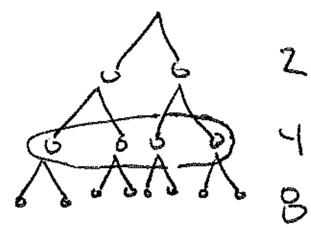
(Ex. 3-3) Max # of nodes at height h will be when the tree is full at that level

Max heap sizes $1+2,\ 1+2+2^2,\ \ldots$ $\displaystyle\sum_{k=0}^{h} 2^k = \left.\frac{2^k}{2-1}\right|_0^{h+1}$

$= 2^{h+1} - 1$

Thus n + h are related in such a way that

at height h max there can be 2^h nodes.



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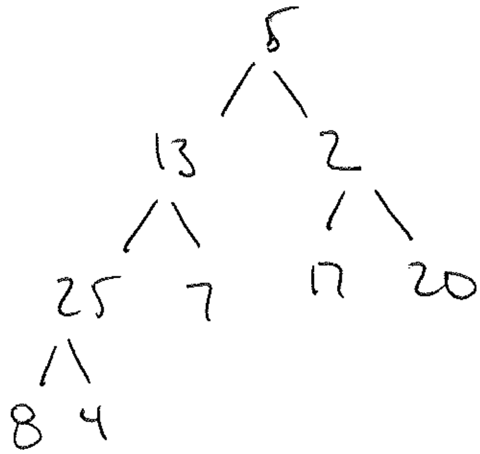
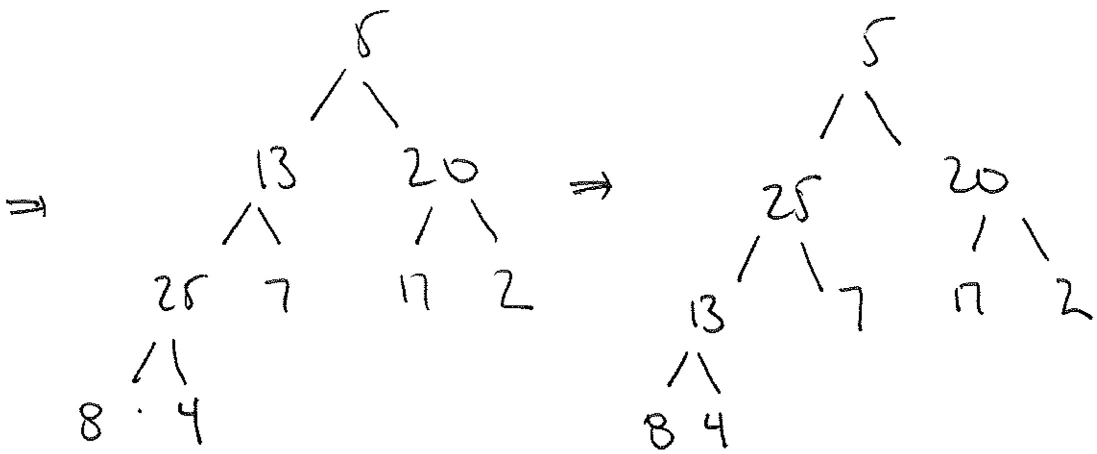
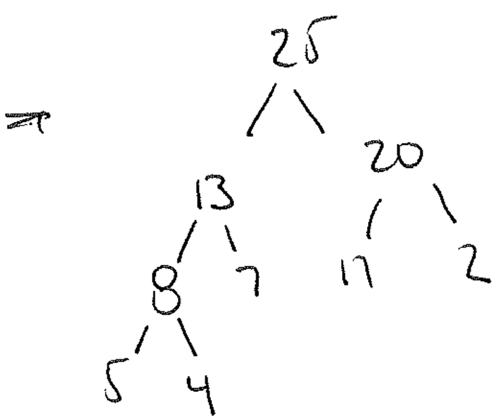
10.4-1



1st call is to build heap:

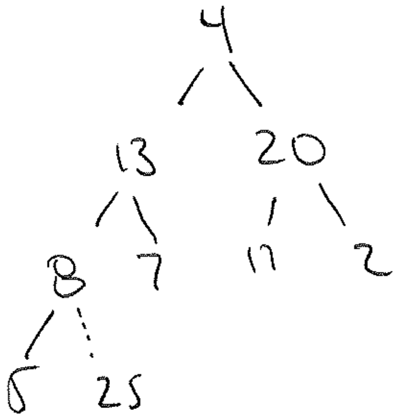




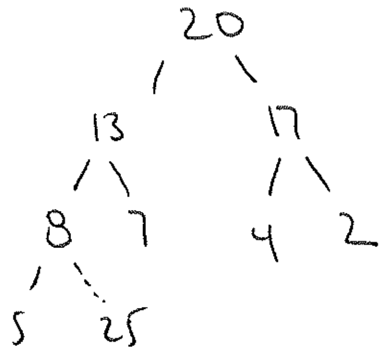
done

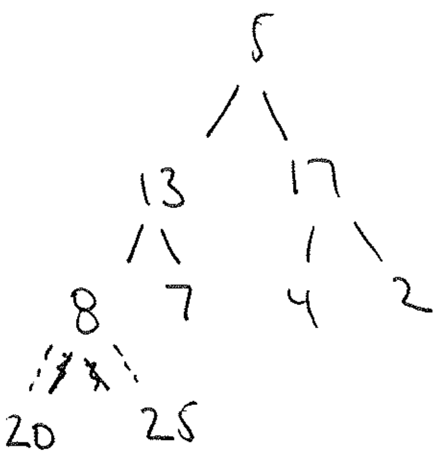
Next pull of at element at position 1 & put at end of

array

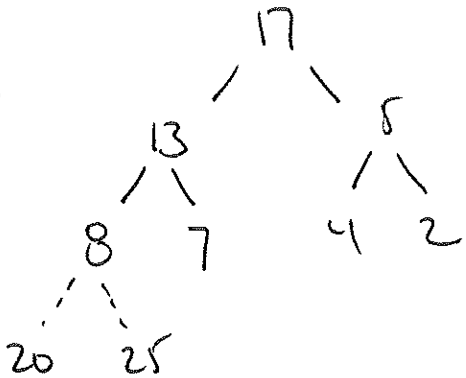


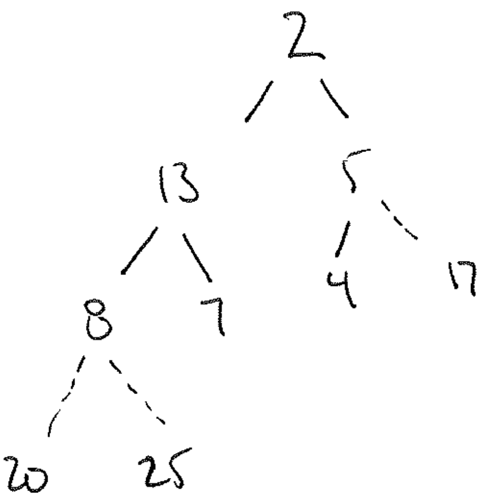
heapify ⇒



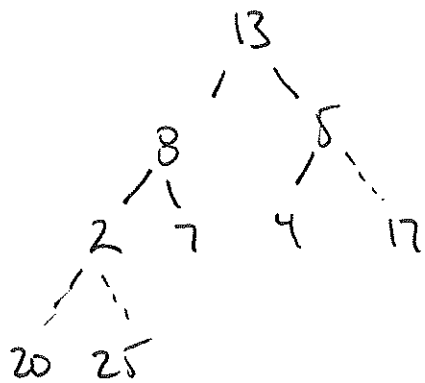


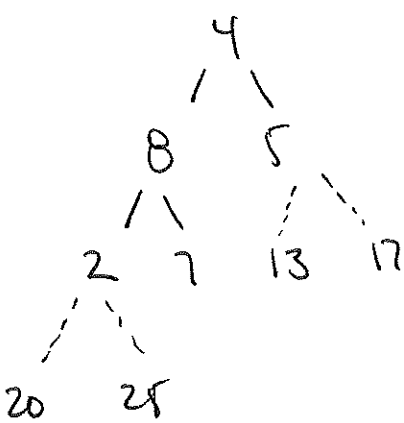
heapify ⇒



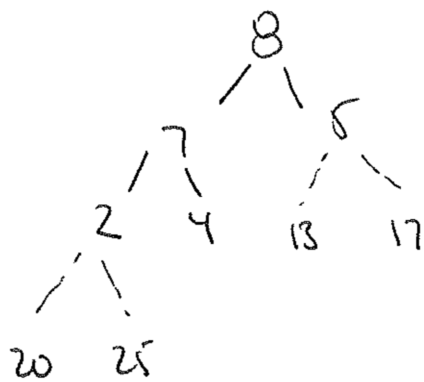


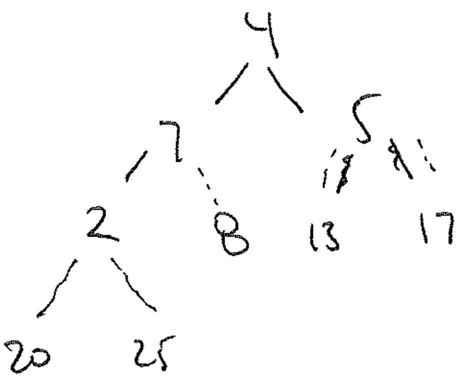
heapify ⇒



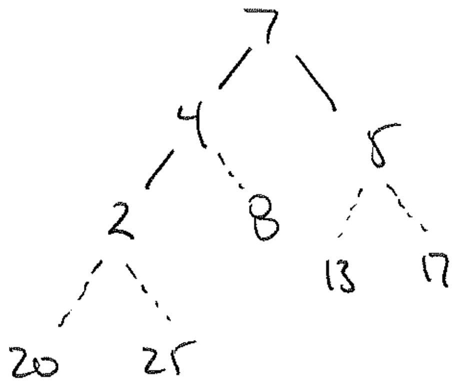


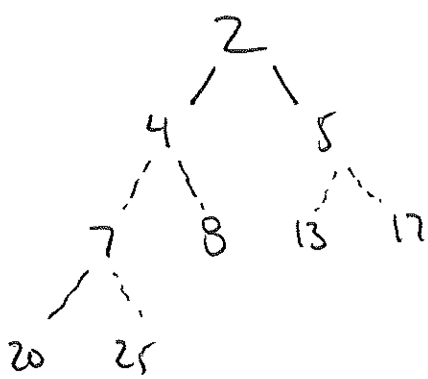
heapify ⇒



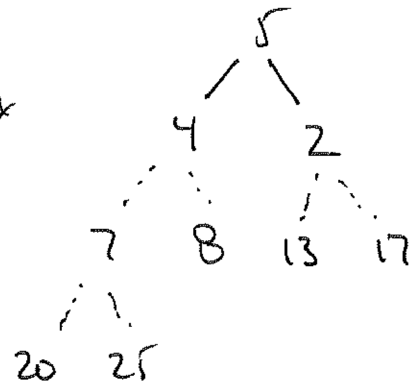


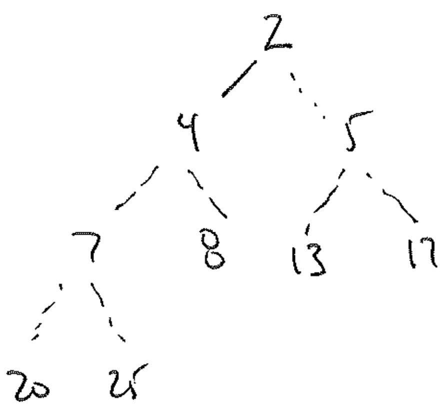
heapify →



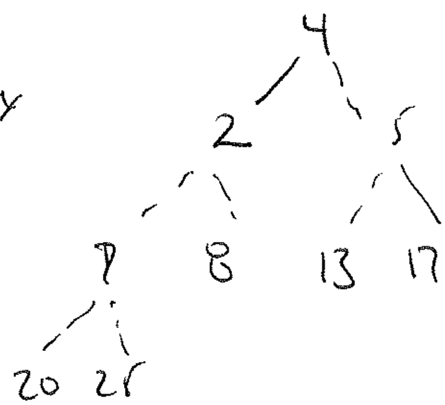


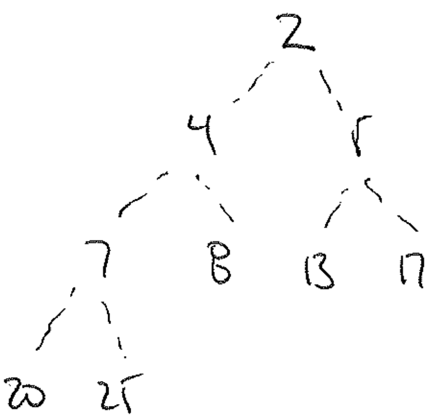
heapify →





heapify →





ordering: 2, 4, 5, 7, 8, 13, 17, 20, 25